MATH 114 Exam #1B Solutions

Given on September 17, 2007

Show all non-trivial calculations.

1. Reduce the rational expressions to lowest terms.

   (a) $\dfrac{x^2-5x-6}{x^2-6x}$

$$\frac{x^2-5x-6}{x^2-6x}=\frac{(x-6)(x+1)}{x(x-6)}=\frac{x+1}{x}$$

   (b) $\dfrac{2x^2-7x-15}{2x^2+7x+6}$

$$\frac{2x^2-7x-15}{2x^2+7x+6}=\frac{(2x+3)(x-5)}{(2x+3)(x+2)}=\frac{x-5}{x+2}$$

2. Multiply or divide the following expressions. Be sure your answer is reduced to simplest form.

   (a) $\dfrac{x^2}{x-5}\cdot\dfrac{x^2-25}{x}$

$$\frac{x^2}{x-5}\cdot\frac{x^2-25}{x}=\frac{x^2}{x-5}\cdot\frac{(x+5)(x-5)}{x}=x(x+5)$$

   (b) $\dfrac{2x+8}{3x-3}\cdot\dfrac{x^2-x}{x^2-16}$

$$\frac{2x+8}{3x-3}\cdot\frac{x^2-x}{x^2-16}=\frac{2(x+4)}{3(x-1)}\cdot\frac{x(x-1)}{(x+4)(x-4)}=\frac{2x}{3(x-4)}$$

   (c) $\dfrac{3a^2}{4b}\div\dfrac{6a}{2b^2}$

$$\frac{3a^2}{4b}\div\frac{6a}{2b^2}=\frac{3a^2}{4b}\cdot\frac{2b^2}{6a}=\frac{ab}{4}$$

(d) $\dfrac{x+3}{x^2+4x-5} \div \dfrac{x^2+4x+3}{x+5}$

$$\begin{aligned}
\frac{x+3}{x^2+4x-5} \div \frac{x^2+4x+3}{x+5} &= \frac{x+3}{x^2+4x-5} \cdot \frac{x+5}{x^2+4x+3} \\
&= \frac{x+3}{(x+5)(x-1)} \cdot \frac{x+5}{(x+3)(x+1)} \\
&= \frac{1}{(x+1)(x-1)}
\end{aligned}$$

3. Add or subtract as indicated. Be sure your answer is reduced to simplest form.

(a) $\dfrac{5}{x} - \dfrac{x}{3}$

$$\frac{5}{x} - \frac{x}{3} = \frac{5}{x} \cdot \frac{3}{3} - \frac{x}{3} \cdot \frac{x}{x} = \frac{15}{3x} - \frac{x^2}{3x} = \frac{15-x^2}{3x}$$

(b) $\dfrac{x^2}{x+3} + \dfrac{3x}{x+3}$

$$\frac{x^2}{x+3} + \frac{3x}{x+3} = \frac{x^2+3x}{x+3} = \frac{x(x+3)}{x+3} = x$$

(c) $\dfrac{x}{x-4} - \dfrac{32}{x^2-16}$

Least common denominator is $x^2 - 16 = (x+4)(x-4)$.

$$\begin{aligned}
\frac{x}{x-4} - \frac{32}{x^2-16} &= \frac{x}{x-4} \cdot \frac{x+4}{x+4} - \frac{32}{(x+4)(x-4)} \\
&= \frac{x(x+4)-32}{(x+4)(x-4)} \\
&= \frac{x^2+4x-32}{(x+4)(x-4)} \\
&= \frac{(x+8)(x-4)}{(x+4)(x-4)} \\
&= \frac{x+8}{x+4}
\end{aligned}$$

(d) $\dfrac{x+6}{x^2-4} - \dfrac{6}{x^2-x-2}$

The least common denominator is $(x+2)(x-2)(x+1)$.

$$\begin{aligned}
\frac{x+6}{x^2-4} - \frac{6}{x^2-x-2} &= \frac{x+6}{(x+2)(x-2)} - \frac{6}{(x-2)(x+1)} \\
&= \frac{x+6}{(x+2)(x-2)} \cdot \frac{x+1}{x+1} - \frac{6}{(x-2)(x+1)} \cdot \frac{x+2}{x+2} \\
&= \frac{(x+6)(x+1) - 6(x+2)}{(x+2)(x-2)(x+1)} \\
&= \frac{x^2+7x+6-6x-12}{(x+2)(x-2)(x+1)} \\
&= \frac{x^2+x-6}{(x+2)(x-2)(x+1)} \\
&= \frac{(x+3)(x-2)}{(x+2)(x-2)(x+1)} \\
&= \frac{x+3}{(x+2)(x+1)}
\end{aligned}$$

4. Solve the equations.

(a) $\dfrac{3x}{4} - \dfrac{5}{2} = 2$

$$\begin{aligned}
\frac{3x}{4} - \frac{5}{2} &= 2 \\
4 \cdot \frac{3x}{4} - 4 \cdot \frac{5}{2} &= 4 \cdot 2 \\
3x - 10 &= 8 \\
3x - 10 + 10 &= 8 + 10 \\
3x &= 18 \\
\frac{3x}{3} &= \frac{18}{3} \\
x &= 6
\end{aligned}$$

(b) $2 + \dfrac{7}{x} = \dfrac{15}{x^2}$

$$2 + \frac{7}{x} = \frac{15}{x^2}$$

$$\begin{aligned}
x^2 \cdot 2 + x^2 \cdot \frac{7}{x} &= x^2 \cdot \frac{15}{x^2} \\
2x^2 + 7x &= 15 \\
2x^2 + 7x - 15 &= 15 - 15 \\
2x^2 + 7x - 15 &= 0 \\
(2x-3)(x+5) &= 0
\end{aligned}$$

$$\begin{aligned}
2x - 3 &= 0 & \text{or} \qquad x + 5 &= 0 \\
2x - 3 + 3 &= 0 + 3 & x + 5 - 5 &= 0 - 5 \\
2x &= 3 & x &= -5 \\
\tfrac{2x}{2} &= \tfrac{3}{2} & & \\
x &= \tfrac{3}{2} & &
\end{aligned}$$

(c) $\dfrac{1}{x+3} + \dfrac{6}{x^2-9} = \dfrac{x}{x-3}$

$$\begin{aligned}
\frac{1}{x+3} + \frac{6}{x^2-9} &= \frac{x}{x-3} \\
\frac{1}{x+3} + \frac{6}{(x+3)(x-3)} &= \frac{x}{x-3} \\
\text{LCD} &= (x+3)(x-3) \\
(x+3)(x-3)\frac{1}{x+3} + (x+3)(x-3)\frac{6}{(x+3)(x-3)} &= (x+3)(x-3)\frac{x}{x-3} \\
(x-3) + 6 &= x(x+3) \\
x + 3 &= x^2 + 3x \\
x + 3 - x - 3 &= x^2 + 3x - x - 3 \\
x^2 + 2x - 3 &= 0 \\
(x+3)(x-1) &= 0
\end{aligned}$$

$$\begin{aligned}
x + 3 &= 0 & \text{or} \qquad x - 1 &= 0 \\
x + 3 - 3 &= 0 - 3 & x - 1 + 1 &= 0 + 1 \\
x &= -3 & x &= 1
\end{aligned}$$

Extraneous

$x = -3$ does *not* work in the original equation. Thus the only solution is $x = 1$.

(d) $\dfrac{x}{x+1} + \dfrac{2}{x-1} = 1$

$$\text{LCD} = (\text{x} + 1)(\text{x} - 1)$$

$$\begin{aligned}
\frac{x}{x+1} + \frac{2}{x-1} &= 1 \\
(x+1)(x-1)\frac{x}{x+1} + (x+1)(x-1)\frac{2}{x-1} &= 1(x+1)(x-1) \\
x(x-1) + 2(x+1) &= (x+1)(x-1) \\
x^2 - x + 2x + 2 &= x^2 - 1 \\
x^2 + x + 2 &= x^2 - 1 \\
x^2 + x + 2 - x^2 &= x^2 - 1 - x^2 \\
x + 2 &= -1 \\
x + 2 - 2 &= -1 - 2 \\
x &= -3
\end{aligned}$$

5. Twice a number decreased by one is the reciprocal of the number. Find the number(s).

a number	means	x
Twice a number	means	$2x$
Twice a number decreased by one	means	$2x - 1$
the reciprocal of the number	means	$1/x$

hence we get the equation

$$2x - 1 = \frac{1}{x}$$

We solve this equation.

$$\begin{aligned}
x \cdot 2x - x \cdot 1 &= x \cdot \frac{1}{x} \\
2x^2 - x &= 1 \\
2x^2 - x - 1 &= 1 - 1 \\
(2x+1)(x-1) &= 0
\end{aligned}$$

$$\begin{aligned} 2x+1 &= 0 \\ 2x+1-1 &= 0-1 \\ 2x &= -1 \\ \tfrac{2x}{2} &= \tfrac{-1}{2} \\ x &= -\tfrac{1}{2} \end{aligned} \qquad \text{or} \qquad \begin{aligned} x-1 &= 0 \\ x-1+1 &= 0+1 \\ x &= 1 \end{aligned}$$

The number is either $-\frac{1}{2}$ or 1.

6. A bathtub has separate hot and cold water faucets. The hot water faucet by itself can fill the tub in 7 minutes. The cold water faucet by itself can fill the tub in 5 minutes. How long will it take to fill the tub with both faucets on all the way?

The key to these problems is to think in terms of the portion of the job done each minute.

	time	tub/min
hot	7	1/7
cold	5	1/5
both	x	$1/x$

Since the portion of the tub filled each minute by both is the sum of the portion filled by the hot and cold faucets, we have the equation

$$\frac{1}{7}+\frac{1}{5}=\frac{1}{x}$$

We solve this equation.

$$\begin{aligned} 35x\cdot\frac{1}{7}+35x\cdot\frac{1}{5} &= 35x\cdot\frac{1}{x} \\ 5x+7x &= 35 \\ 12x &= 35 \\ \frac{12x}{12} &= \frac{35}{12} \\ x &= \frac{35}{12} \end{aligned}$$

It would take $\frac{35}{12}\approx 2.9$ minutes to fill the tub.

7. Simplify the complex fractions.

(a) $\dfrac{\dfrac{2x}{x-5}}{\dfrac{4x^2}{x^2-25}}$

$$\begin{aligned}\frac{\dfrac{2x}{x-5}}{\dfrac{4x^2}{x^2-25}} &= \frac{2x}{x-5}\cdot\frac{x^2-25}{4x^2}\\ &= \frac{2x}{x-5}\cdot\frac{(x+5)(x-5)}{4x^2}\\ &= \frac{x+5}{2x}\end{aligned}$$

(b) $\dfrac{\dfrac{4}{y}-1}{4-\dfrac{1}{2y}}$

We multiply the numerator and denominator by the LCD of the fractions contained within the large fraction.

$$\begin{aligned}\frac{\dfrac{4}{y}-1}{4-\dfrac{1}{2y}} &= \frac{\dfrac{4}{y}-1}{4-\dfrac{1}{2y}}\cdot\frac{2y}{2y}\\ &= \frac{2y\cdot\dfrac{4}{y}-2y\cdot 1}{2y\cdot 4-2y\cdot\dfrac{1}{2y}}\\ &= \frac{8-2y}{8y-1}\end{aligned}$$

This fraction cannot be reduced, as we cannot *factor* the same number or expression from both numerator and denominator.

8. If a 5 pound bag of peanuts cost \$12.50, how much should a 3 pound bag of the same peanuts cost?

   Let x = the cost of a 3 pound bag. If the 3 pound bag is sold at a proportional price to the 5 pound bag, we have

$$\begin{aligned} \frac{3}{x} &= \frac{5}{12.50} \\ 3(12.50) &= 5x \\ \frac{3(12.50)}{5} &= \frac{5x}{5} \\ x &= 7.50 \end{aligned}$$

   A 3 pound bag should cost \$7.50.

9. Solve the equations.

   (a) $3(2x - 3) + 2x = 4x + 15$

$$\begin{aligned} 3(2x-3)+2x &= 4x+15 \\ 6x-9+2x &= 4x+15 \\ 8x-9 &= 4x+15 \\ 8x-9-4x &= 4x+15-4x \\ 4x-9 &= 15 \\ 4x-9+9 &= 15+9 \\ 4x &= 24 \\ \frac{4x}{4} &= \frac{24}{4} \\ x &= 6 \end{aligned}$$

   (b) $3(4x - 5) = 5(2x + 4) + 2x$

$$\begin{aligned} 3(4x-5) &= 5(2x+4)+2x \\ 12x-15 &= 10x+20+2x \\ 12x-15 &= 12x+20 \\ 12x-15-12x &= 12x+20-12x \\ -15 &= 20 \end{aligned}$$

   Since this last equation simply cannot be true for any value of x, their is no solution.

(c) $(2x-3)(x+5)=7$

$$\begin{aligned}(2x-3)(x+5) &= 7\\ 2x^2+7x-15 &= 7\\ 2x^2+7x-15-7 &= 7-7\\ 2x^2+7x-22 &= 0\\ (2x+11)(x-2) &= 0\end{aligned}$$

$$\begin{aligned}2x+11 &= 0 &\text{or}\quad x-2 &= 0\\ 2x+11-11 &= 0-11 & x-2+2 &= 0+2\\ 2x &= -11 & x &= 2\\ \tfrac{2x}{2} &= \tfrac{-11}{2} & &\\ x &= -\tfrac{11}{2} & &\end{aligned}$$

(d) $12x^2=4x$

$$\begin{aligned}12x^2 &= 4x\\ 12x^2-4x &= 4x-4x\\ 4x(3x-1) &= 0\end{aligned}$$

$$\begin{aligned}x &= 0 \quad\text{or} & 3x-1 &= 0\\ & & 3x-1+1 &= 0+1\\ & & 3x &= 1\\ & & \tfrac{3x}{3} &= \tfrac{1}{2}\\ & & x &= \tfrac{1}{2}\end{aligned}$$

(e) $|4x-3|=9$

$$\begin{aligned}4x-3 &= 9 &\text{or}\quad 4x-3 &= -9\\ 4x-3+3 &= 9+3 &\text{or}\quad 4x-3+3 &= -9+3\\ 4x &= 12 & 4x &= -6\\ \tfrac{4x}{4} &= \tfrac{12}{4} & \tfrac{4x}{4} &= \tfrac{-6}{4}\\ x &= 3 &\text{or}\quad x &= -\tfrac{3}{2}\end{aligned}$$

(f) $|3x-5|-10=7$

$$\begin{aligned} |3x-5|-10 &= 7 \\ |3x-5|-10+10 &= 7+10 \\ |3x-5| &= 17 \end{aligned}$$

$$\begin{aligned} 3x-5 &= 17 & \text{or} \quad 3x-5 &= -17 \\ 3x-5+5 &= 17+5 & 3x-5+5 &= -17+5 \\ 3x &= 22 & 3x &= -12 \\ \tfrac{3x}{3} &= \tfrac{22}{3} & \tfrac{3x}{3} &= \tfrac{-12}{3} \\ x &= \tfrac{22}{3} & \text{or} \quad x &= -4 \end{aligned}$$

(g) $|x+5|=|2x-3|$

$$\begin{aligned} x+5 &= 2x-3 & \text{or} \quad x+5 &= -(2x-3) \\ x+5-x &= 2x-3-x & x+5 &= -2x+3 \\ 5 &= x-3 & x+5+2x &= -2x+3+2x \\ 5+3 &= x-3+3 & 3x+5 &= 3 \\ x &= 8 & 3x+5-5 &= 3-5 \\ & & 3x &= -2 \\ & & \tfrac{3x}{3} &= \tfrac{-2}{3} \\ & & x &= -\tfrac{2}{3} \end{aligned}$$