MATH 114 Exam #3B Solutions
Given on November 6, 2007

1. For the functions $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, find

   (a) $(f + g)(2)$

$$(f+g)(2) = f(2) + G(2) = [3(2) - 5] + [(2)^2 + 2] = 6 - 5 + 4 + 2 = 7$$

   (b) $(fg)(x)$

$$(fg)(x) = f(x) \times g(x) = (3x - 5)(x^2 + 2) = 3x^3 + 6x - 5x^2 - 10$$

   (c) $(f \circ g)(1)$

$$(f \circ g)(1) = f[g(1)] == f[(1)^2 + 2] = f(3) = 3(3) - 5 = 4$$

   (d) $(g \circ f)(x)$

$$\begin{aligned} (g \circ f)(x) &= g[f(x)] \\ &= g[3x - 5] \\ &= (3x - 5)^2 + 2 \\ &= (3x)^2 - 2(3x)(5) + 5^2 + 2 \\ &= 9x^2 - 30x + 25 + 2 \\ &= 9x^2 - 30x + 27 \end{aligned}$$

2. z varies directly with the square of x and inversely with y. If $z = 12$ when $x = 2$ and $y = 1$, find z when $x = 1$ and $y = 2$.

   z varies directly with the square of x and inversely with y.
   $z \quad = \quad k \quad x^2 \quad /y$

$$z = \frac{kx^2}{y}$$

   Then if $z = 12$ when $x = 2$ and $y = 1$,

$$\begin{aligned} 12 &= \frac{k(2)^2}{1} \\ 12 &= 4k \\ \frac{12}{4} &= \frac{4k}{4} \\ k &= 3 \\ z &= \frac{3x^2}{y} \end{aligned}$$

   So for $x = 1$ and $y = 2$,

$$z = \frac{3(1)^2}{2} = \frac{3}{2}$$

3. Simplify the following expressions. Assume all bases represent positive numbers.

(a) $(-8)^{2/3}$

$$(-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$$

(b) $\left(\frac{16}{9}\right)^{-3/2}$

$$\left(\frac{16}{9}\right)^{-3/2} = \left(\frac{9}{16}\right)^{3/2} = \frac{9^{3/2}}{16^{3/2}} = \frac{\left(\sqrt{9}\right)^3}{\left(\sqrt{16}\right)^3} = \frac{3^3}{4^3} = \frac{27}{64}$$

(c) $a^{1/2}(a^{1/3}b^{2/3})^{3/2}$

$$\begin{aligned} a^{1/2}(a^{1/3}b^{2/3})^{3/2} &= a^{1/2}\left(a^{1/3}\right)^{3/2}\left(b^{2/3}\right)^{3/2} \\ &= a^{1/2}a^{\frac{1}{3}\times\frac{3}{2}}b^{\frac{2}{3}\times\frac{3}{2}} \\ &= a^{1/2}a^{1/2}b \\ &= a^{\frac{1}{2}+\frac{1}{2}}b \\ &= ab \end{aligned}$$

(d) $\frac{x^{1/2}y^{-3}}{xy^{3/2}}$

$$\begin{aligned} \frac{x^{1/2}y^{-3}}{xy^{3/2}} &= \frac{x^{1/2}}{x} \times \frac{y^{-3}}{y^{3/2}} \\ &= x^{\frac{1}{2}-1}y^{-3-\frac{3}{2}} \\ &= x^{\frac{1}{2}-\frac{2}{2}}y^{-\frac{6}{2}-\frac{3}{2}} \\ &= x^{-1/2}y^{-9/2} \\ &= \frac{1}{x^{1/2}y^{9/2}} \end{aligned}$$

(e) $\sqrt[3]{27x^6y^3}$

$$\sqrt[3]{27x^6y^3} = \sqrt[3]{3^3\left(x^2\right)^3y^3} = \sqrt[3]{(3x^2y)^3} = 3x^2y$$

(f) $\sqrt{12x^3y^4}$

$$\sqrt{12x^3y^4} = \sqrt{4x^2y^4}\sqrt{3x} = 2xy^2\sqrt{3x}$$

(g) $\sqrt[4]{32x^5y^{10}}$

$$\sqrt[4]{32x^5y^{10}} = \sqrt[4]{16x^4y^8}\sqrt[4]{2xy^2} = 2xy^2\sqrt[4]{2xy^2}$$

4. Multiply and simplify. Assume $x > 0$.

(a) $3x^{3/2}(2x^{1/2} + x)$

$$\begin{aligned} 3x^{3/2}(2x^{1/2} + x) &= \left(3x^{3/2}\right)\left(2x^{1/2}\right) + \left(3x^{3/2}\right)(x) \\ &= 6x^{\frac{3}{2}+\frac{1}{2}} + 3x^{\frac{3}{2}+1} \\ &= 6x^{\frac{4}{2}} + 3x^{\frac{3}{2}+\frac{2}{2}} \\ &= 6x^2 + 3x^{5/2} \end{aligned}$$

(b) $(s^{1/3}-5)^2$

$$\begin{aligned}(s^{1/3}-5)^2 &= \left(s^{1/3}\right)^2 - 2\left(s^{1/3}\right)(5) + 5^2\\ &= s^{\frac{1}{3}\times 2} - 10s^{1/3} + 25\\ &= s^{2/3} - 10s^{1/3} + 25\end{aligned}$$

(c) $\sqrt{5}(\sqrt{10}-5\sqrt{2})$

$$\begin{aligned}\sqrt{5}(\sqrt{10}-5\sqrt{2}) &= \sqrt{10}\sqrt{5} - 5\sqrt{2}\sqrt{5}\\ &= \sqrt{50} - 5\sqrt{10}\\ &= \sqrt{25}\sqrt{2} - 5\sqrt{10}\\ &= 5\sqrt{2} - 5\sqrt{10}\end{aligned}$$

(d) $(3\sqrt{2}-5)(2\sqrt{2}+3)$

$$\begin{aligned}(3\sqrt{2}-5)(2\sqrt{2}+3) &= (3\sqrt{2})(2\sqrt{2}) + 3(3\sqrt{2}) - 5(2\sqrt{2}) - 5(3)\\ &= 6(\sqrt{2})^2 + 9\sqrt{2} - 10\sqrt{2} - 15\\ &= 6(2) - \sqrt{2} - 15\\ &= -\sqrt{2} - 3\end{aligned}$$

(e) $(2\sqrt{2x}-\sqrt{3})(2\sqrt{2x}+\sqrt{3})$
Use the difference of squares formula $(a-b)(a+b) = a^2 - b^2$

$$\begin{aligned}(2\sqrt{2x}-\sqrt{3})(2\sqrt{2x}+\sqrt{3}) &= \left(2\sqrt{2x}\right)^2 - \left(\sqrt{3}\right)^2\\ &= 4(2x) - 3\\ &= 8x - 3\end{aligned}$$

5. Divide $\dfrac{24x^{3/2} - 16x^{5/4}}{4x^{3/4}}$

$$\begin{aligned}\frac{24x^{3/2} - 16x^{5/4}}{4x^{3/4}} &= \frac{24x^{3/2}}{4x^{3/4}} - \frac{16x^{5/4}}{4x^{3/4}}\\ &= \frac{24}{4} \times \frac{x^{3/2}}{x^{3/4}} - \frac{16}{4} \times \frac{x^{5/4}}{x^{3/4}}\\ &= 6x^{\frac{3}{2}-\frac{3}{4}} - 4x^{\frac{5}{4}-\frac{3}{4}}\\ &= 6x^{\frac{6}{4}-\frac{3}{4}} - 4x^{2/4}\\ &= 6x^{3/4} - 4x^{1/2}\end{aligned}$$

6. Rationalize the denominators. Assume $x, y > 0$.

(a) $\sqrt{\dfrac{2}{3x}}$

$$\sqrt{\frac{2}{3x}} = \frac{\sqrt{2}}{\sqrt{3x}} \times \frac{\sqrt{3x}}{\sqrt{3x}} = \frac{\sqrt{2}\sqrt{3x}}{\left(\sqrt{3x}\right)^2} = \frac{\sqrt{6x}}{3x}$$

(b) $\dfrac{3y}{\sqrt[3]{9x}}$

$$\frac{3y}{\sqrt[3]{9x}} = \frac{3y}{\sqrt[3]{9x}} \times \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \frac{3y\sqrt[3]{3x^2}}{\sqrt[3]{27x^3}} = \frac{3y\sqrt[3]{3x^2}}{3x} = \frac{y\sqrt[3]{3x^2}}{x}$$

(c) $\dfrac{2x}{\sqrt{x}+2}$

$$\begin{aligned}\frac{2x}{\sqrt{x}+2} &= \frac{2x}{\sqrt{x}+2} \times \frac{\sqrt{x}-2}{\sqrt{x}-2} \\ &= \frac{2x\sqrt{x}-2x(2)}{\left(\sqrt{x}\right)^2 - 2^2} \\ &= \frac{2x\sqrt{x}-4x}{x-4}\end{aligned}$$

(d) $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

$$\begin{aligned}\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} &= \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} \times \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} \\ &= \frac{\left(\sqrt{x}\right)^2 + 2\sqrt{x}\sqrt{y} + \left(\sqrt{y}\right)^2}{\left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2} \\ &= \frac{x+2\sqrt{xy}+y}{x-y}\end{aligned}$$

In the numerator, we have made use of the formula $(a+b)^2 = a^2 + 2ab + b^2$. In the denominator, we use the formula $(a-b)(a+b) = a^2 - b^2$.

7. Simplify and combine like terms.
$3x\sqrt[3]{4xy^4} - 2y\sqrt[3]{32x^4y}$

$$\begin{aligned}3x\sqrt[3]{4xy^4} - 2y\sqrt[3]{32x^4y} &= 3x\sqrt[3]{y^3}\sqrt[3]{4xy} - 2y\sqrt[3]{8x^3}\sqrt[3]{4xy} \\ &= 3xy\sqrt[3]{4xy} - 2y(2x)\sqrt[3]{4xy} \\ &= 3xy\sqrt[3]{4xy} - 4xy\sqrt[3]{4xy} \\ &= -xy\sqrt[3]{4xy}\end{aligned}$$

8. Solve the equations.

(a) $\sqrt{x+3} - 5 = 2$

$$\begin{aligned}\sqrt{x+3} - 5 &= 2 \\ \sqrt{x+3} - 5 + 5 &= 2 + 5 \\ \sqrt{x+3} &= 7 \\ \left(\sqrt{x+3}\right)^2 &= 7^2 \\ x + 3 &= 49 \\ x + 3 - 3 &= 49 - 3 \\ x &= 46\end{aligned}$$

You should check this in the original equation to insure that it works.

(b) $\sqrt[3]{3x-2} = 4$

$$\begin{aligned} \sqrt[3]{3x-2} &= 4 \\ \left(\sqrt[3]{3x-2}\right)^3 &= 4^3 \\ 3x-2 &= 64 \\ 3x-2+2 &= 64+2 \\ 3x &= 66 \\ \frac{3x}{3} &= \frac{66}{3} \\ x &= 22 \end{aligned}$$

Again, check this in the original equation to insure that it works.

(c) $\sqrt{2x+14} - x = 3$

$$\begin{aligned} \sqrt{2x+14} - x &= 3 \\ \sqrt{2x+14} - x + x &= 3 + x \\ \sqrt{2x+14} &= 3 + x \\ \left(\sqrt{2x+14}\right)^2 &= (x+3)^2 \\ 2x+14 &= x^2 + 2(3)(x) + 3^2 \\ 2x+14 &= x^2 + 6x + 9 \\ 2x+14-2x-14 &= x^2+6x+9-2x-14 \\ x^2+4x-5 &= 0 \\ (x+5)(x-1) &= 0 \\ x+5=0 \quad &\text{or} \quad x-1=0 \\ x+5-5=0-5 \quad & \quad x-1+1=0+1 \\ x=-5 \quad & \quad \boxed{\text{x=1}} \\ \text{Extraneous} \quad & \end{aligned}$$

$x = 1$ satisfies the original equation, but $x = -5$ does not.

(d) $\sqrt{x+25} = \sqrt{x-2} + 3$

$$\begin{aligned}
\sqrt{x+25} &= \sqrt{x-2} + 3 \\
\left(\sqrt{x+25}\right)^2 &= \left(\sqrt{x-2} + 3\right)^2 \\
x + 25 &= \left(\sqrt{x-2}\right)^2 + 2\sqrt{x-2}(3) + 3^2
\end{aligned}$$

Here we have used the formula $(a+b)^2 = a^2 + 2ab + b^2$ on the right side. Note that we *cannot* just square the two terms, i.e. $\left(\sqrt{x-2}\right)^2 + 3^2$. Continuing,

$$\begin{aligned}
x + 25 &= x - 2 + 6\sqrt{x-2} + 9 \\
x + 25 &= x + 7 + 6\sqrt{x-2} \\
x + 25 - x - 7 &= x + 7 - x - 7 + 6\sqrt{x-2} \\
18 &= 6\sqrt{x-2} \\
\frac{18}{6} &= \frac{6\sqrt{x-2}}{6} \\
\sqrt{x-2} &= 3 \\
\left(\sqrt{x-2}\right)^2 &= 3^2 \\
x - 2 &= 9 \\
x - 2 + 2 &= 9 + 2 \\
x &= 11
\end{aligned}$$

Again, check this in the original equation to insure that it works.