

Show all non-trivial calculations.

1. Find all real and complex solutions to the equations:

$$(a) \frac{3}{2}(2x - 5) + \frac{1}{2} = 5x + 3$$

$$\begin{aligned} \frac{3}{2}(2x - 5) + \frac{1}{2} &= 5x + 3 \\ 2 \times \frac{3}{2}(2x - 5) + 2 \times \frac{1}{2} &= 2 \times (5x + 3) \\ 3(2x - 5) + 1 &= 2(5x + 3) \\ 6x - 14 &= 10x + 6 \\ 6x - 14 - 6x &= 10x + 6 - 6x \\ -14 &= 4x + 6 \\ -14 - 6 &= 4x + 6 - 6 \\ 4x &= -20 \\ \frac{4x}{4} &= \frac{-20}{4} \\ x &= -5 \end{aligned}$$

$$(b) (3x - 2)(x + 7) - 10x = (x + 1)^2$$

$$\begin{aligned} (3x - 2)(x + 7) - 10x &= (x + 1)^2 \\ 3x^2 + 9x - 14 &= x^2 + 2x + 1 \\ 3x^2 + 9x - 14 - (x^2 + 2x + 1) &= x^2 + 2x + 1 - (x^2 + 2x + 1) \\ 2x^2 + 7x - 15 &= 0 \\ (2x - 3)(x + 5) &= 0 \\ 2x - 3 &= 0 \quad \text{or} \quad x + 5 = 0 \\ 2x - 3 + 3 &= 0 + 3 \quad x + 5 - 5 = 0 - 5 \\ 2x &= 3 \quad x &= -5 \\ 2x/2 &= 3/2 \\ x &= 3/2 \end{aligned}$$

$$(c) 4x^2 - 3x - 11 = 0$$

We use the quadratic formula, with $a = 4$, $b = -3$, $c = -11$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned}
&= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-11)}}{2(4)} \\
&= \frac{3 \pm \sqrt{185}}{8}
\end{aligned}$$

(d) $3x^2 + 5 = 2x$

First, we must have zero on the right side.

$$\begin{aligned}
3x^2 + 5 &= 2x \\
3x^2 + 5 - 2x &= 2x - 2x \\
3x^2 - 2x + 5 &= 0
\end{aligned}$$

We use the quadratic formula, with $a = 3, b = -2, c = 5$.

$$\begin{aligned}
x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)} \\
&= \frac{2 \pm \sqrt{-56}}{6} \\
&= \frac{2 \pm \sqrt{4}\sqrt{-1}\sqrt{14}}{6} \\
&= \frac{2}{6} \pm \frac{2i\sqrt{14}}{6} \\
&= \frac{1}{3} \pm \frac{\sqrt{14}}{3}i
\end{aligned}$$

(e) $x^4 - 13x^2 + 36 = 0$

$$\begin{aligned}
x^4 - 13x^2 + 36 &= 0 \\
(x^2 - 4)(x^2 - 9) &= 0
\end{aligned}$$

$$\begin{array}{lcl}
x^2 - 4 = 0 & \text{or} & x^2 - 9 = 0 \\
x^2 - 4 + 4 = 0 + 4 & & x^2 - 9 + 9 = 0 + 9 \\
x^2 = 4 & & x^2 = 9 \\
x = \pm\sqrt{4} & & x = \pm\sqrt{9} \\
x = \pm 2 & & x = \pm 3
\end{array}$$

(f) $x^3 + 3x^2 - 16x - 48 = 0$

$$\begin{aligned}
x^3 + 3x^2 - 16x - 48 &= 0 \\
x^2(x + 3) - 16(x + 3) &= 0 \\
(x + 3)(x^2 - 16) &= 0
\end{aligned}$$

$$\begin{array}{rcl}
x + 3 = 0 & \text{or} & x^2 - 16 = 0 \\
x + 3 - 3 = 0 - 3 & & x^2 - 16 + 16 = 0 + 16 \\
x = -3 & & x^2 = 16 \\
& & x = \pm\sqrt{16} \\
& & x = \pm 4
\end{array}$$

(g) $\sqrt{12 - 2x} + 2 = x$

$$\begin{aligned}
\sqrt{12 - 2x} + 2 &= x \\
\sqrt{12 - 2x} + 2 - 2 &= x - 2 \\
(\sqrt{12 - 2x})^2 &= (x - 2)^2 \\
12 - 2x &= x^2 - 4x + 4 \\
12 - 2x - 12 + 2x &= x^2 - 4x + 4 - 12 + 2x \\
x^2 - 2x - 8 &= 0 \\
(x - 4)(x + 2) &= 0
\end{aligned}$$

$$\begin{array}{rcl}
x - 4 = 0 & \text{or} & x + 2 = 0 \\
x - 4 + 4 = 0 + 4 & & x + 2 - 2 = 0 - 2 \\
x = 4 & & x = -2
\end{array}$$

$x = -2$ does not work in the original equation. It is extraneous. The only valid solution is $x = 4$.

(h) $\sqrt{2x + 3} - \sqrt{x + 1} = 1$

$$\begin{aligned}
\sqrt{2x + 3} - \sqrt{x + 1} &= 1 \\
\sqrt{2x + 3} - \sqrt{x + 1} + \sqrt{x + 1} &= 1 + \sqrt{x + 1} \\
\sqrt{2x + 3} &= 1 + \sqrt{x + 1} \\
(\sqrt{2x + 3})^2 &= (1 + \sqrt{x + 1})^2 \\
2x + 3 &= 1 + 2\sqrt{x + 1} + (\sqrt{x + 1})^2 \\
2x + 3 &= 1 + 2\sqrt{x + 1} + x + 1 \\
2x + 3 - x - 2 &= 1 + 2\sqrt{x + 1} + x + 1 - x - 2 \\
x + 1 &= 2\sqrt{x + 1} \\
(x + 1)^2 &= (2\sqrt{x + 1})^2 \\
x^2 + 2x + 1 &= 4(x + 1) \\
x^2 + 2x + 1 - 4x - 4 &= 4x + 4 - 4x - 4 \\
x^2 - 2x - 3 &= 0 \\
(x - 3)(x + 1) &= 0
\end{aligned}$$

$$\begin{array}{rcl}
 x - 3 & = & 0 \quad \text{or} \quad x + 1 = 0 \\
 x - 3 + 3 & = & 0 + 3 \quad x + 1 - 1 = 0 - 1 \\
 x & = & 3 \quad \quad \quad x = -1
 \end{array}$$

You should verify that both solutions work.

(i) $|x^2 - x - 4| + 3 = 5$

$$\begin{array}{rcl}
 |x^2 - x - 4| + 3 & = & 5 \\
 |x^2 - x - 4| + 3 - 3 & = & 5 - 3 \\
 |x^2 - x - 4| & = & 2 \\
 x^2 - x - 4 & = & 2 \quad \text{or} \quad x^2 - x - 4 = -2 \\
 x^2 - x - 4 - 2 & = & 2 - 2 \quad x^2 - x - 4 + 2 = -2 + 2 \\
 x^2 - x - 6 & = & 0 \quad \quad \quad x^2 - x - 2 = 0 \\
 (x - 3)(x + 2) & = & 0 \quad \quad \quad (x - 2)(x + 1) = 0 \\
 x - 3 = 0 \quad \text{or} \quad x + 2 = 0 & \text{or} & x - 2 = 0 \quad \text{or} \quad x + 1 = 0 \\
 x = 3 \quad \quad \quad x = -2 & & x = 2 \quad \quad \quad x = -1
 \end{array}$$

2. Solve the following inequalities. State your solution in interval notation.

(a) $-2 < \frac{5 - 3x}{2} < 7$

$$\begin{array}{rcl}
 -2 \times 2 & < & \frac{5 - 3x}{2} \times 2 < 7 \times 2 \\
 -4 & < & 5 - 3x < 14 \\
 -4 - 5 & < & 5 - 3x - 5 < 14 - 5 \\
 -9 & < & -3x < 9 \\
 \frac{-9}{-3} & > & \frac{-3x}{-3} > \frac{9}{-3} \\
 3 & > & x > -3 \\
 -3 & < & x < 3
 \end{array}$$

In interval notation, this is $(-3, 3)$.

(b) $|2x - 3| + 5 > 12$

$$\begin{array}{rcl}
 |2x - 3| + 5 & > & 12 \\
 |2x - 3| + 5 - 5 & > & 12 - 5 \\
 |2x - 3| & > & 7 \\
 2x - 3 & < & -7 \quad \text{or} \quad 2x - 3 > 7 \\
 2x - 3 + 3 & < & -7 + 3 \quad 2x - 3 + 3 > 7 + 3 \\
 2x & < & -4 \quad \quad \quad 2x > 10 \\
 2x/2 & < & -4/2 \quad \quad \quad 2x/2 > 10/2 \\
 x & < & -2 \quad \quad \quad x > 5
 \end{array}$$

In interval notation, this is $(-\infty, -2) \cup (5, \infty)$.

(c) $|5x + 1| \leq 14$

$$\begin{array}{rcl} -14 & \leq & 5x + 1 \leq 14 \\ -14 - 1 & \leq & 5x + 1 - 1 \leq 14 - 1 \\ -15 & \leq & 5x \leq 13 \\ -15/5 & \leq & 5x/5 \leq 13/5 \\ -3 & \leq & x \leq 13/5 \end{array}$$

In interval notation, this is $\left[-3, \frac{13}{5}\right]$.

3. For the complex numbers below, perform the indicated operation and express the answer in standard form.

(a) $(4 + 3i)(3 - 2i)$

$$(4 + 3i)(3 - 2i) = 12 + i - 6i^2 = 12 + i - 6(-1) = 18 + i$$

(b) i^{37}

Dividing 37 by 4 gives us a remainder of 1, hence

$$i^{37} = i^{4 \times 9 + 1} = (i^4)^9 i^1 = 1^9 i^1 = i^1 = i$$

(c) $\frac{3 + i}{5 - 3i}$

$$\begin{aligned} \frac{3 + i}{5 - 3i} &= \frac{3 + i}{5 - 3i} \frac{5 + 3i}{5 + 3i} = \frac{15 + 14i + 3i^2}{5^2 - (3i)^2} = \frac{15 + 14i + 3(-1)}{25 - 9(-1)} = \frac{12 + 14i}{34} \\ &= \frac{2(6 + 7i)}{2(17)} = \frac{6 + 7i}{17} = \frac{6}{17} + \frac{7}{17}i \end{aligned}$$

4. A water tank can be filled by an inlet pipe in 7 hours. It can be drained in 10 hours. If the tank is empty and both the inlet and outlet are opened fully, how long will it take to fill the tank?

Let x = the time taken to fill the tank with both valves open. The key is to think in terms of the portion of the job done per unit of time.

	time	tank/hour
inlet	7	1/7
outlet	10	1/10
both	x	1/x

The amount of water in the tank after one hour is the amount that comes in minus the amount that goes out. Hence

$$\begin{aligned} \frac{1}{7} - \frac{1}{10} &= \frac{1}{x} \\ 70x \cdot \frac{1}{7} - 70x \cdot \frac{1}{10} &= 70x \cdot \frac{1}{x} \\ 10x - 7x &= 70 \\ 3x &= 70 \\ \frac{3x}{3} &= \frac{70}{3} \\ x &= \frac{70}{3} \end{aligned}$$

It would take 23 hours and 20 minutes to fill the tank.

5. How much pure water should be added to 2 liters of 15% salt solution to make an 8% salt solution?

The key to mixture problems like this is to think in terms of the actual quantity of the important ingredient (salt).

	quantity	percentage	amount of salt
original mixture	2 liters	15	$.15 \times 2 = .3$ liters
water	x	0	0
final mixture	$2 + x$	8	$.08 \times (2 + x) = .16 + .08x$ liters

The amount of salt in the final mixture is the sum of the amounts of salt in the original mixtures. Hence

$$\begin{aligned} .3 + 0 &= .16 + .08x \\ .3 - .16 &= .16 + .08x - .16 \\ .14 &= .08x \\ \frac{.14}{.08} &= \frac{.08x}{.08} \\ x &= 1.75 \end{aligned}$$

1.75 liters of pure water need to be added.

6. Dan sets off to walk the 2 miles to work at a pace of 3 miles per hour. After he leaves his son Chris notices that he forgot his briefcase. Chris straps the briefcase to his bicycle and leaves 20 minutes after Dan to catch him. If Chris bicycles at 8 miles/hour, how long will it take him to catch his father?

Let x = the time it takes Chris to catch his father. Note that 20 minutes is $1/3$ of an hour. We use the fact that distance = rate \times time to complete the table below.

	distance	rate	time
Dan	$3(x + 1/3)$	3	$x + 1/3$
Chris	$8x$	8	x

At the point where Chris catches his father, they have both traveled exactly the same distance, so

$$\begin{aligned}
 3\left(x + \frac{1}{3}\right) &= 8x \\
 3x + 1 &= 8x \\
 3x + 1 - 3x &= 8x - 3x \\
 5x &= 1 \\
 \frac{5x}{5} &= \frac{1}{5} \\
 x &= \frac{1}{5}
 \end{aligned}$$

It takes Chris $1/5$ of an hour = 12 minutes to catch his father.

7. For the points $(-2, 5)$ and $(3, -3)$.

(a) Find the distance between the points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[3 - (-2)]^2 + (-3 - 5)^2} = \sqrt{25 + 64} = \sqrt{89}$$

(b) Find the midpoint.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 3}{2}, \frac{5 + (-3)}{2}\right) = \left(\frac{1}{2}, 1\right)$$

(c) Find the slope of the line containing the points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{3 - (-2)} = -\frac{8}{5}$$

8. For the following equations, find the intercepts (x and y), and test for symmetry about the x -axis, y -axis, and origin.

(a) $y^2 = 9 - x^2$

• x -intercept(s): Let $y = 0$ and solve for x .

$$\begin{aligned}
 (0)^2 &= 9 - x^2 \\
 0 + x^2 &= 9 - x^2 + x^2 \\
 x^2 &= 9 \\
 x &= \pm 3
 \end{aligned}$$

- y -intercept(s): Let $x = 0$ and solve for y .

$$\begin{aligned}y^2 &= 9 - (0)^2 \\y^2 &= 9 \\y &= \pm 3\end{aligned}$$

- x -axis symmetry: Replace y with $-y$.

$$\begin{aligned}(-y)^2 &= 9 - x^2 \\y^2 &= 9 - x^2\end{aligned}$$

Since we can obtain the original equation by simplifying, we have symmetry about the x -axis.

- y -axis symmetry: Replace x with $-x$.

$$\begin{aligned}y^2 &= 9 - (-x)^2 \\y^2 &= 9 - x^2\end{aligned}$$

Since we can obtain the original equation by simplifying, we have symmetry about the y -axis.

- origin symmetry: Replace x with $-x$ and y with $-y$.

$$\begin{aligned}(-y)^2 &= 9 - (-x)^2 \\y^2 &= 9 - x^2\end{aligned}$$

Since we can obtain the original equation by simplifying, we have symmetry about the origin. This last step was actually unnecessary, since it is impossible to have exactly two of the three types of symmetry. Hence once we found symmetry about both the x and y axes, symmetry about the origin was assured.

Note: I will expect to see symmetry tests such as this on the midterm exam. Graphing the equation on the calculator is *not* sufficient. $y = (x - .001)^2$ looks symmetric about the y -axis, but the algebraic test shows us that it is not.

(b) $y^2 = x + 9$

- x -intercept(s):

$$\begin{aligned}(0)^2 &= x + 9 \\x + 9 - 9 &= 0 - 9 \\x &= -9\end{aligned}$$

- y -intercept(s):

$$\begin{aligned}y^2 &= (0) + 9 \\y^2 &= 9 \\y &= \pm 3\end{aligned}$$

- x -axis symmetry:

$$\begin{aligned}(-y)^2 &= x + 9 \\ y^2 &= x + 9\end{aligned}$$

Since we can obtain the original equation by simplifying, we have symmetry about the x -axis.

- y -axis symmetry:

$$\begin{aligned}y^2 &= (-x) + 9 \\ y^2 &= 9 - x\end{aligned}$$

Since we cannot obtain the original equation by simplifying, we do not have symmetry about the y -axis.

- origin symmetry: Since this would give us exactly two of the three types of symmetry, which is impossible, we know that the graph is not symmetric about the origin.

9. Write the general form of the equation of a circle with center at $(3, -1)$ and radius 2.

$$\begin{aligned}(x - 3)^2 + [y - (-1)]^2 &= (2)^2 \\ x^2 - 6x + 9 + (y + 1)^2 &= 4 \\ x^2 - 6x + 9 + y^2 + 2y + 1 &= 4 \\ x^2 - 6x + 9 + y^2 + 2y + 1 - 4 &= 4 - 4 \\ x^2 + y^2 - 6x + 2y + 6 &= 0\end{aligned}$$

10. Find the center and radius of the circle whose equation is $x^2 + y^2 + 2x - 4y - 4 = 0$

We must complete the square for both x and y .

$$\begin{aligned}x^2 + y^2 + 2x - 4y - 4 &= 0 \\ x^2 + y^2 + 2x - 4y - 4 + 4 &= 0 + 4 \\ x^2 + 2x + \underline{\quad} + y^2 - 4y + \underline{\quad} &= 4 + \underline{\quad} + \underline{\quad} \\ x^2 + 2x + \left(\frac{1}{2}(2)\right)^2 + y^2 - 4y + \left(\frac{1}{2}(-4)\right)^2 &= 4 + \left(\frac{1}{2}(2)\right)^2 + \left(\frac{1}{2}(-4)\right)^2 \\ x^2 + 2x + 1 + y^2 - 4y + 4 &= 4 + 1 + 4 \\ (x + 1)^2 + (y - 2)^2 &= 9\end{aligned}$$

Comparing this with the standard form $(x - h)^2 + (y - k)^2 = R^2$, we have $h = -1$, $k = 2$, $R = 3$. The circle has center at $(-1, 2)$ and radius 3.

11. Find an equation of the line having the following characteristics.

- (a) Slope of -3 and y -intercept of 2.

We have $m = -3$ and $b = 2$. Using these in the slope-intercept form $y = mx + b$, we have

$$y = -3x + 2$$

(b) Slope of 2 containing the point $(3, -1)$

We use the point-slope formula $y - y_1 = m(x - x_1)$.

$$\begin{aligned}y - (-1) &= 2(x - 3) \\y + 1 &= 2x - 6 \\y + 1 - 1 &= 2x - 6 - 1 \\y &= 2x - 7\end{aligned}$$

(c) Containing the points $(2, 5)$ and $(-2, -3)$

First, we calculate the slope.

$$m = \frac{5 - (-3)}{2 - (-2)} = \frac{8}{4} = 2$$

We use this and one of the given points in the point-slope formula.

$$\begin{aligned}y - 5 &= 2(x - 2) \\y - 5 &= 2x - 4 \\y - 5 + 5 &= 2x - 4 + 5 \\y &= 2x + 1\end{aligned}$$

(d) The vertical line through the point $(5, -2)$

A vertical line is the set of all points having a common x -coordinate. Its equation simply specifies that common x -coordinate. In this case the equation is

$$x = 5$$

(e) Perpendicular to the line $2x - y = 7$, containing the point $(3, 2)$.

First, we find the slope of the given line by putting it into slope-intercept form.

$$\begin{aligned}2x - y &= 7 \\2x - y - 2x &= 7 - 2x \\-y &= -2x + 7 \\y &= 2x - 7\end{aligned}$$

The slope is the coefficient of x , so $m_2 = 2$. The line whose equation we seek is perpendicular to this. Its slope is given by

$$m_1 = -\frac{1}{m_2} = -\frac{1}{2}$$

We use this and the given point to find the equation.

$$y - 2 = -\frac{1}{2}(x - 3)$$

$$\begin{aligned}
y - 2 &= -\frac{1}{2}x + \frac{3}{2} \\
y - 2 + 2 &= -\frac{1}{2}x + \frac{3}{2} + 2 \\
y &= -\frac{1}{2}x + \frac{7}{2}
\end{aligned}$$

12. Find the slope and y -intercept of the line $3x + 2y = 8$

We put the equation into slope-intercept form.

$$\begin{aligned}
3x + 2y &= 8 \\
3x + 2y - 3x &= 8 - 3x \\
2y &= -3x + 8 \\
\frac{2y}{2} &= \frac{-3x}{2} + \frac{8}{2} \\
y &= -\frac{3}{2}x + 2
\end{aligned}$$

Then the slope is $m = -3/2$ and the y -intercept is $b = 2$.

13. z varies directly with the square of x and inversely with the square root of y . If $z = 3$ when $x = 1$ and $y = 4$, find z when $x = 2$ and $y = 9$.

$$\begin{aligned}
z &= k \frac{x^2}{\sqrt{y}} \\
3 &= k \frac{(1)^2}{\sqrt{4}} \\
3 &= \frac{1}{2}k \\
2 \times 3 &= 2 \times \frac{1}{2}k \\
k &= 6
\end{aligned}$$

The general formula is

$$z = \frac{6x^2}{\sqrt{y}}$$

. When $x = 2$ and $y = 9$,

$$z = \frac{6(2)^2}{\sqrt{9}} = \frac{24}{3} = 8$$

14. Of the following sets, circle those which represent function(s).

(a) $\boxed{(0, 2), (1, 4), (2, -3), (3, 1)}$

(b) $\boxed{(1, 3), (2, 3), (3, 2), (4, 0)}$

- (c) $(1, 3), (2, 0), (2, 5), (3, 0)$

For a function, any value of x in the domain must give us one and only one value of y . (c) is not a function, since when $x = 2$, y is either 0 or 5. (a) and (b) are functions.

15. For the functions $f(x) = x^2 - 1$, and $g(x) = x + 1$ find

- (a) $f(0)$

$$f(0) = (0)^2 - 1 = -1$$

- (b) $g(-3)$

$$g(-3) = (-3) + 1 = -2$$

- (c) $f(2a)$

$$f(2a) = (2a)^2 - 1 = 4a^2 - 1$$

- (d) $g(x + 2)$

$$g(x + 2) = (x + 2) + 1 = x + 3$$

- (e) $(f + g)(2)$

$$(f + g)(2) = f(2) + g(2) = [(2)^2 - 1] + [(2) + 1] = 4 - 1 + 2 + 1 = 6$$

- (f) $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x) = (x^2 - 1)(x + 1) = x^3 + x^2 - x - 1$$

- (g) $(f/g)(x)$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 1}{x + 1} = \frac{(x + 1)(x - 1)}{x + 1} = x - 1 \quad ; \quad x \neq -1$$

16. Find the domain of the function.

(a) $f(x) = \frac{3x - 1}{x^2 + 4}$

There are two things that concern us here. Denominators cannot be zero and we cannot take an even root of a negative number. Since there are no radicals, the second is not an issue. Notice that the denominator cannot be zero in this case anyway, since the equation $x^2 + 4 = 0$ has no real solutions. Hence the domain is all real numbers.

(b) $f(x) = \frac{x + 1}{x^2 - 9}$

We need to remove any zeroes of the denominator. Set the denominator equal to zero and solve.

$$\begin{aligned}x^2 - 9 &= 0 \\x^2 &= 9 \\x &= \pm 3\end{aligned}$$

The domain is all real number except ± 3 . $\{x|x \neq \pm 3\}$

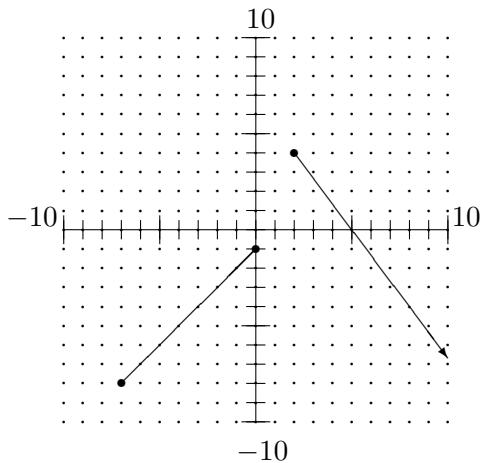
(c) $f(x) = \sqrt{2x - 3}$

We have an even root, so we must insure that the radicand is not negative.

$$\begin{aligned} 2x - 3 &\geq 0 \\ 2x - 3 + 3 &\geq 0 + 3 \\ 2x &\geq 3 \\ \frac{2x}{2} &\geq \frac{3}{2} \\ x &\geq \frac{3}{2} \end{aligned}$$

In interval notation, this would be $[\frac{3}{2}, \infty)$.

17. For the function $f(x)$ graphed below, do the following:



- (a) State the domain: $[-7, 0] \cup [2, \infty)$
 (b) State the range: $(-\infty, 4]$
 (c) State the x -intercept(s): $(5, 0)$
 (d) State the y -intercept: $(0, -1)$
 (e) State the increasing intervals: $(-7, 0)$
 (f) State the decreasing intervals: $(2, \infty)$
 (g) Find $f(-6)$: -7
 (h) If $f(x) = -4$, find x : $x = -3$ or $x = 8$

Determine whether the following functions are even, odd, or neither. Justify your answer.

(a) $f(x) = \frac{x^2 - 1}{x}$

$$f(-x) = \frac{(-x)^2 - 1}{(-x)} = \frac{x^2 - 1}{-x} = -\frac{x^2 - 1}{x}$$

Since $f(-x) = -f(x)$, the function is odd.

(b) $f(x) = 2x^3 - 3x - 4$

$$f(-x) = 2(-x)^3 - 3(-x) - 4 = -2x^3 + 3x - 4 = -(2x^3 - 3x + 4)$$

$f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$. The function is neither even nor odd.

(c) $f(x) = \sqrt{x^2 - 1}$

$$f(-x) = \sqrt{(-x)^2 - 1} = \sqrt{x^2 - 1}$$

Since $f(-x) = f(x)$, the function is even.

18. For the function $f(x) = \begin{cases} x & \text{if } x < -2 \\ 2x + 2 & \text{if } -2 \leq x \leq 2 \\ -3x & \text{if } x > 2 \end{cases}$

(a) Find $f(-5)$

Since $-5 < -2$, we evaluate using the first line. $f(-5) = -5$.

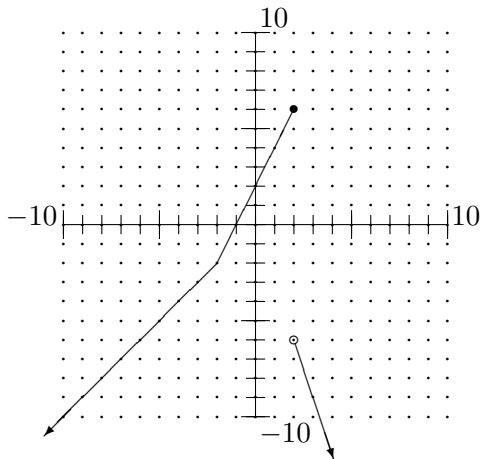
(b) Find $f(0)$

Since $-2 \leq 0 \leq 2$, we evaluate using the second line. $f(0) = 2(0) + 2 = 2$

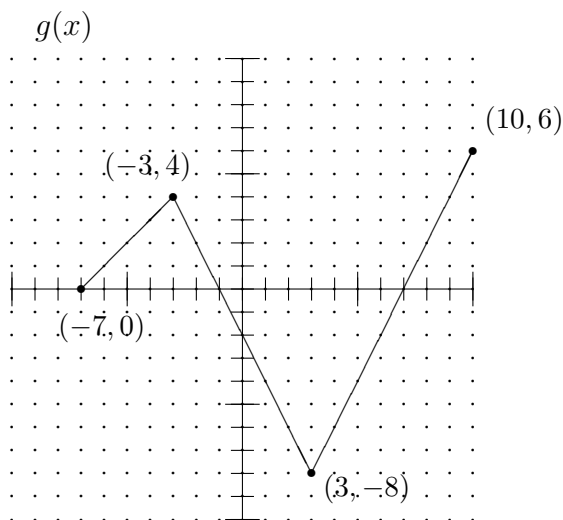
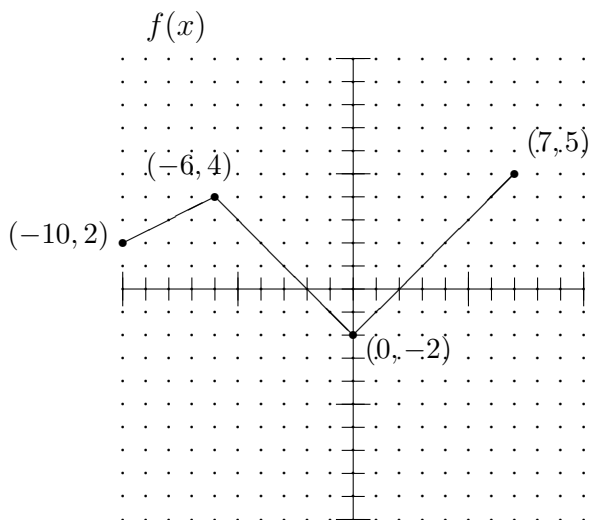
(c) Find $f(2)$

Since $-2 \leq 2 \leq 2$, we evaluate using the second line. $f(2) = 2(2) + 2 = 6$

(d) Graph the function.



19. Shown below is a function $f(x)$. Graph the related function $g(x) = 2f(x - 3) - 4$



The function $g(x)$ is $f(x)$ shifted to the right by 3 units, stretched along the y -axis by a factor of 2, and shifted down the y -axis by 4 units, in that order. We first determine where the labeled points will end up.

point	right by 3	y-stretch by 2	down by 4
$(-10, 2)$	$(-7, 2)$	$(-7, 4)$	$(-7, 0)$
$(-6, 4)$	$(-3, 4)$	$(-3, 8)$	$(-3, 4)$
$(0, -2)$	$(3, -2)$	$(3, -4)$	$(3, -8)$
$(7, 5)$	$(10, 5)$	$(10, 10)$	$(10, 6)$

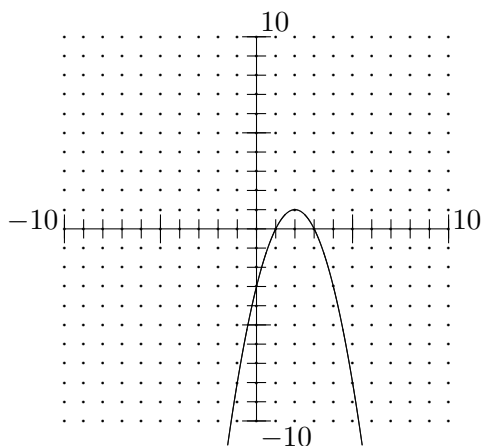
Now we connect those points with line segments, and we have the transformed graph.

20. Find the average rate of change of the function $f(x) = 2x^2 - 3x$ from $x = 2$ to $x = 5$.

$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(2)}{5 - 2} = \frac{[2(5)^2 - 3(5)] - [2(2)^2 - 3(2)]}{3} = \frac{35 - 2}{3} = \frac{33}{3} = 11$$

21. For the quadratic functions given below, find the y -intercepts, any x -intercepts, the vertex and graph the function.

(a) $f(x) = -x^2 + 4x - 3$



The x -coordinate of the vertex is given by

$$h = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$$

The y -coordinate of the vertex is given by

$$k = f(h) = f(2) = -(2)^2 + 4(2) - 3 = 1$$

The y -intercept is

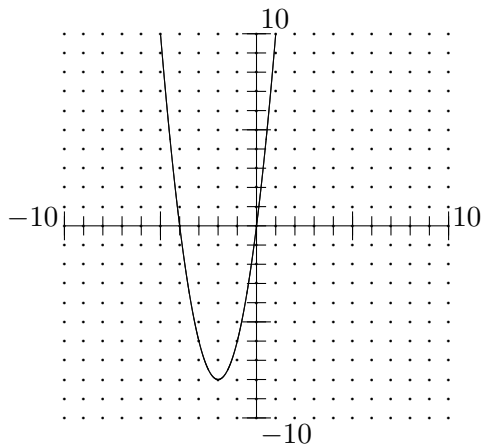
$$f(0) = -(0)^2 + 4(0) - 3 = -3$$

The x -intercepts are found by solving the equation $f(x) = 0$.

$$\begin{aligned} -x^2 + 4x - 3 &= 0 \\ -(-x^2 + 4x - 3) &= -0 \\ x^2 - 4x + 3 &= 0 \\ (x - 3)(x - 1) &= 0 \\ x - 3 = 0 &\text{ or } x - 1 = 0 \\ x = 3 &\quad x = 1 \end{aligned}$$

Vertex (2, 1) y -intercept (0, -3) x -intercepts (1, 0), (3, 0)

(b) $g(x) = 2x^2 + 8x$



vertex:

$$h = -\frac{b}{2a} = -\frac{8}{2(2)} = -2 \quad k = g(-2) = 2(-2)^2 + 8(-2) = -8$$

y -intercept:

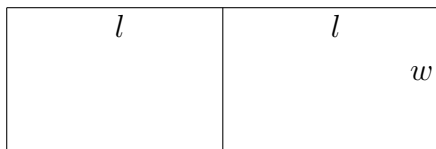
$$f(0) = 2(0)^2 + 8(0) = 0$$

x -intercepts:

$$\begin{aligned} 2x^2 + 8x &= 0 \\ 2x(x + 4) &= 0 \\ x = 0 &\text{ or } x + 4 = 0 \\ &\quad x = -4 \end{aligned}$$

Vertex (-2, -8) y -intercept (0, 0) x -intercepts (0, 0), (-4, 0)

22. A gardener wishes to fence off two equal plots using 100 ft of fence. To make the most out of the fence, she intends to fence off one large rectangle, and then run a single length of fence down the middle to divide it into two smaller rectangles, as shown below. What should the dimensions (length and width) of each of the smaller rectangles be in order to obtain the maximum area?



There are four sections of fence of length l and three sections of length w . Since there are 100ft of fence,

$$4l + 3w = 100$$

Solving this equation for w , we have

$$\begin{aligned} 4l + 3w &= 100 \\ 4l + 3w - 4l &= 100 - 4l \\ 3w &= 100 - 4l \\ w &= \frac{100 - 4l}{3} \end{aligned}$$

Then the area of the large rectangle is

$$A(l) = 2lw = 2l \left(\frac{100 - 4l}{3} \right) = -\frac{8}{3}l^2 + \frac{200}{3}l$$

This is a quadratic function which opens downward, hence it has a maximum value at its vertex. We need only find the x -coordinate of the vertex.

$$h = -\frac{b}{2a} = -\frac{\frac{200}{3}}{2\left(-\frac{8}{3}\right)} = \frac{200}{16} = 12.5\text{ft}$$

$$w = \frac{100 - 4(12.5)}{3} = \frac{50}{3}\text{ft}$$

Each of the small rectangles will be $12\frac{1}{2}$ ft by $16\frac{2}{3}$ ft

23. Find an equation of a 4th degree polynomial with a root of multiplicity 2 at $x = 3$ and roots of multiplicity 1 at $x = -2$ and $x = 0$.

Each root c corresponds to a factor $x - c$, so one such polynomial would be

$$f(x) = (x - 3)^2(x - (-2))(x - 0) = x(x - 3)^2(x + 2)$$

Expanding this (not required) we get

$$f(x) = x^4 - 4x^3 - 3x^2 + 18x$$

24. Which of the following are factors of the polynomial $x^4 - 2x^3 - 14x^2 - 2x - 15$? Justify your answer.

(a) $(x + 2)$

If $x + 2 = x - (-2)$ is a factor of the polynomial, then $x = -2$ must be a root.

$$f(-2) = (-2)^4 - 2(-2)^3 - 14(-2)^2 - 2(-2) - 15 = -35 \neq 0$$

Hence $x + 2$ is not a factor. (Note: Using a calculator to evaluate these functions is recommended.)

(b) $(x + 3)$

$$x + 3 = x - (-3) \quad f(-3) = 0$$

Since -3 is a root, $x + 3$ is a factor.

(c) $(x - 1)$

$$f(1) = -32 \neq 0$$

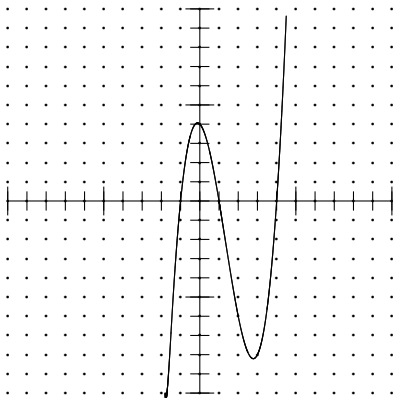
So $x - 1$ is not a factor.

(d) $(x - 5)$

$$f(5) = 0$$

So $x - 5$ is a factor.

25. For the following function, find the intercepts and sketch the graph. *Label* (number) the graph appropriately. $f(x) = x^3 - 4x^2 - x + 4$



y-intercept:

$$f(0) = (0)^3 - 4(0)^2 - (0) + 4 = 4$$

x-intercepts: Solve the equation $f(x) = 0$.

$$\begin{aligned} x^3 - 4x^2 - x + 4 &= 0 \\ x^2(x - 4) - (x - 4) &= 0 \\ (x - 4)(x^2 - 1) &= 0 \\ (x - 4)(x + 1)(x - 1) &= 0 \end{aligned}$$

$$\begin{array}{rcl} x - 4 = 0 & \text{or} & x + 1 = 0 & \text{or} & x - 1 = 0 \\ x = 4 & & x = -1 & & x = 1 \end{array}$$

x -intercepts $(4, 0), (1, 0), (-1, 0)$ y -intercepts $(4, 0)$

26. For the function $f(x) = \frac{2x^2}{x^2 - 9}$ find

- any vertical asymptotes.

Vertical asymptotes occur at zeroes of the denominator.

$$\begin{array}{rcl} x^2 - 9 & = & 0 \\ x^2 & = & 9 \\ x & = & \pm 3 \end{array}$$

There are vertical asymptotes at $x = 3$ and $x = -3$.

- any horizontal or oblique asymptotes

Since the degree of the numerator is the same as the degree of the denominator, there is a horizontal asymptote at $y = \frac{a}{b}$ where a is the leading coefficient of the numerator and b is the leading coefficient of the denominator. In this case, we have a horizontal asymptote at

$$y = \frac{2}{1} = 2$$

- x -intercepts

x -intercepts are zeroes of the numerator.

$$\begin{array}{rcl} 2x^2 & = & 0 \\ x & = & 0 \end{array}$$

- y -intercept

$$f(0) = \frac{2(0)^2}{(0)^2 - 9} = 0$$

The origin is both an x -intercept and y -intercept.

- positive intervals

We use the asymptotes and x -intercepts to chop the x -axis into the intervals $(-\infty, -3)$, $(-3, 0)$, $(0, 3)$, $(3, \infty)$. We then select a test point within each interval and use it to determine whether the function is positive or negative on that interval.

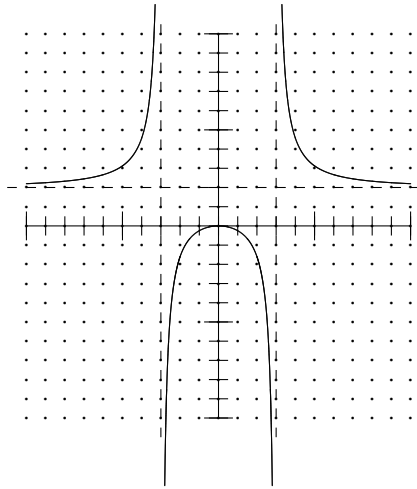
interval	test point	function value	pos/neg
$(-\infty, -3)$	-4	$f(-4) \approx 4.6$	pos
$(-3, 0)$	-1	$f(-1) = -\frac{1}{4}$	neg
$(0, 3)$	1	$f(1) = -\frac{1}{4}$	neg
$(3, \infty)$	4	$f(4) \approx 4.6$	pos

So the positive intervals are $(-\infty, -3)$ and $(3, \infty)$.

- negative intervals

From the table above, the function is negative on the interval $(-3, 3)$.

- sketch the graph



27. For the function $f(x) = \frac{2x - 3}{x^2 - 4}$ find

- any vertical asymptotes.

$$\begin{aligned} x^2 - 4 &= 0 \\ (x + 2)(x - 2) &= 0 \\ x + 2 = 0 &\text{ or } x - 2 = 0 \\ x = -2 &\quad x = 2 \end{aligned}$$

- any horizontal or oblique asymptotes

Since the degree of the numerator is less than the degree of the denominator, the x -axis is a horizontal asymptote.

- x -intercepts

$$\begin{aligned} 2x - 3 &= 0 \\ 2x - 3 + 3 &= 0 + 3 \\ 2x &= 3 \\ \frac{2x}{2} &= \frac{3}{2} \\ x &= \frac{3}{2} \end{aligned}$$

- y -intercept

$$f(0) = \frac{2(0) - 3}{(0)^2 - 4} = \frac{-3}{-4} = \frac{3}{4}$$

- positive intervals

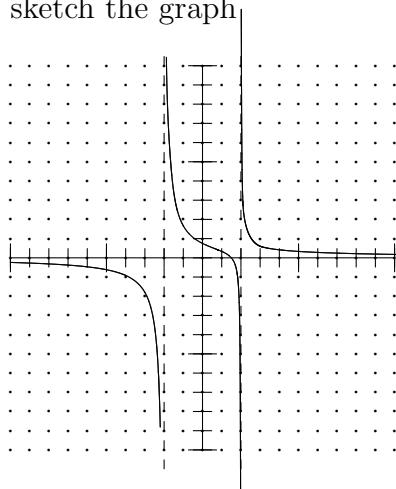
interval	test point	function value	pos/neg
$(-\infty, -2)$	-3	$f(-3) = -1.8$	neg
$(-2, \frac{3}{2})$	0	$f(0) = \frac{3}{4}$	pos
$(\frac{3}{2}, 2)$	$\frac{7}{4}$	$f(\frac{7}{4}) \approx -0.53$	neg
$(2, \infty)$	3	$f(3) = 0.6$	pos

Positive intervals are $(-2, \frac{3}{2})$ and $(2, \infty)$

- negative intervals

$(-\infty, -2)$ and $(\frac{3}{2}, 2)$.

- sketch the graph



28. Solve the inequalities.

(a) $2x^3 + 3x \leq 5x^2$

First, we need zero on the right side.

$$\begin{aligned}
 2x^3 + 3x &\leq 5x^2 \\
 2x^3 + 3x - 5x^2 &\leq 5x^2 - 5x^2 \\
 2x^3 - 5x^2 + 3x &\leq 0
 \end{aligned}$$

Now we find the solutions to the corresponding equation.

$$\begin{aligned}
 2x^3 - 5x^2 + 3x &= 0 \\
 x(2x^2 - 5x + 3) &= 0 \\
 x(2x - 3)(x - 1) &= 0
 \end{aligned}$$

$$\begin{array}{rcl}
x = 0 & \text{or} & 2x - 3 = 0 & \text{or} & x - 1 = 0 \\
x = 0 & & 2x - 3 + 3 = 0 + 3 & & x - 1 + 1 = 0 + 1 \\
& & 2x = 3 & & x = 1 \\
& & 2x/2 = 3/2 & & \\
& & x = 3/2 & &
\end{array}$$

Now we chop the number line into pieces at these points to obtain the intervals

$$(-\infty, 0), (0, 1), (1, 3/2), (3/2, \infty)$$

On each of these intervals, the expression $2x^3 - 5x^2 + 3x$ is either always positive or always negative. We simply need to choose a test point inside each interval and evaluate $2x^3 - 5x^2 + 3x$ to see which it is.

interval	$(-\infty, 0)$	$(0, 1)$	$(1, 3/2)$	$(3/2, \infty)$
test point	-1	1/2	5/4	2
value of $2x^3 - 5x^2 + 3x$	-10	1/2	-5/32	2

The inequality is solved whenever the value of $2x^3 - 5x^2 + 3x$ is less than or equal to 0, hence the solution set is

$$(-\infty, 0] \cup [1, 3/2]$$

(b) $\frac{x^2 - 4}{x^2 - 9} > 0$

We find the zeroes of the numerator and the zeroes of the denominator.

$$\begin{array}{rcl}
x^2 - 4 = 0 & \text{or} & x^2 - 9 = 0 \\
x^2 - 4 + 4 = 0 + 4 & & x^2 - 9 + 9 = 0 + 9 \\
x^2 = 4 & & x^2 = 9 \\
x = \pm 2 & & x = \pm 3
\end{array}$$

Now we chop the number line into pieces at these points to obtain the intervals

$$(-\infty, -3), (-3, -2), (-2, 2), (2, 3), (3, \infty)$$

We need to evaluate the expression $\frac{x^2 - 4}{x^2 - 9}$ on each of these intervals.

interval	$(-\infty, -3)$	$(-3, -2)$	$(-2, 2)$	$(2, 3)$	$(3, \infty)$
test point	-4	-2.5	0	2.5	4
value of $\frac{x^2 - 4}{x^2 - 9}$	1.71	-0.82	4/9	-0.82	1.71

The inequality is solved whenever the value of $\frac{x^2 - 4}{x^2 - 9}$ is greater than 0, hence the solution set is

$$(-\infty, -3) \cup (-2, 2) \cup (3, \infty)$$

29. For the function $f(x) = \frac{2x^2 - 3}{x^2 + 1}$ and $g(x) = \sqrt{x - 1}$, do the following:

(a) Find $(g \circ f)(3)$

(b) Find $(f \circ g)(x)$

(c) State the domain of $(f \circ g)(x)$

30. For the function $f(x) = \frac{5-x}{1-x}$, find the inverse function $f^{-1}(x)$.

$$\begin{aligned}y &= \frac{5-x}{1-x} \\y \cdot (1-x) &= \frac{5-x}{1-x} \cdot (1-x) \\y - yx &= 5-x \\y - yx + x - y &= 5-x + x - y \\x - yx &= 5-y \\x(1-y) &= 5-y \\\frac{x(1-y)}{1-y} &= \frac{5-y}{1-y} \\x &= \frac{5-y}{1-y}\end{aligned}$$

Interchanging x and y , we have

$$y = \frac{5-x}{1-x}$$

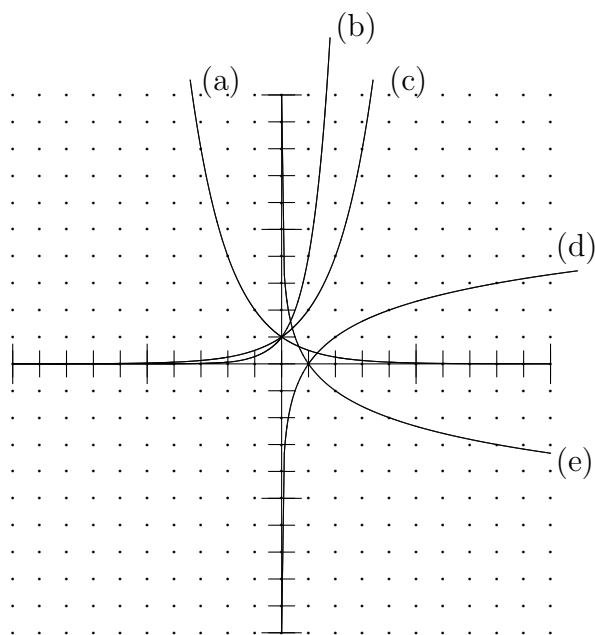
So the inverse function is

$$f^{-1}(x) = \frac{5-x}{1-x}$$

Note that in this particular case, $f^{-1}(x) = f(x)$. You can verify this by showing that

$$f(f(x)) = x$$

31. Match the functions to their graphs by writing the letter of the graph in front of the appropriate function:



- d $f(x) = \log_2 x$
 e $f(x) = \log_{\frac{1}{2}} x$
 c $f(x) = 2^x$
 b $f(x) = 4^x$
 a $f(x) = \left(\frac{1}{2}\right)^x$

32. Simplify the following.

(a) $\log_3 \frac{1}{27}$

$$\log_3 \frac{1}{27} = \log_3 3^{-3} = -3 \log_3 3 = -3(1) = -3$$

(b) $4^{\log_4 5x}$

$$4^{\log_4 5x} = 5x$$

b^u and $\log_b u$ are inverse functions. They undo one another.

(c) $\log_2 4^{3x}$

$$\log_2 4^{3x} = \log_2 (2^2)^{3x} = \log_2 2^{6x} = 6x \log_2 2 = 6x(1) = 6x$$

33. Express $\log x^3 - 2 \log x + 2 \log xy$ as a single logarithm.

$$\log x^3 - 2 \log x + 2 \log xy = \log x^3 - \log x^2 + \log (xy)^2 = \log \frac{x^3(xy)^2}{x^2} = \log x^3 y^2$$

34. Express $\ln \frac{x^3 y}{z^2}$ in terms of $\ln x$, $\ln y$ and $\ln z$.

$$\ln \frac{x^3 y}{z^2} = \ln x^3 + \ln y - \ln z^2 = 3 \ln x + \ln y - 2 \ln z$$

35. Solve the equations for x . If your answer is irrational, use your calculator to approximate it to 3 decimal places.

(a) $\log_2 x = 3$

$$\log_2 x = 3 \quad \Rightarrow \quad x = 2^3 = 8$$

(b) $3^{3x} = 9^{2x-1}$

$$\begin{aligned} 3^{3x} &= 9^{2x-1} \\ 3^{3x} &= (3^2)^{2x-1} \\ 3^{3x} &= 3^{2(2x-1)} \\ 3x &= 2(2x-1) \\ 3x &= 4x-2 \\ 3x-4x &= 4x-2-4x \\ -x &= -2 \\ x &= 2 \end{aligned}$$

(c) $5^x = 10$

$$\begin{aligned} 5^x &= 10 \\ \log 5^x &= \log 10 \\ x \log 5 &= \log 10 \\ \frac{x \log 5}{\log 5} &= \frac{\log 10}{\log 5} \\ x &= \frac{\log 10}{\log 5} \\ x &\approx 1.431 \end{aligned}$$

We could have used natural logarithms just as well.

(d) $\log x - \log 4 = \log 2$

$$\begin{aligned} \log x - \log 4 &= \log 2 \\ \log \frac{x}{4} &= \log 2 \\ \frac{x}{4} &= 2 \\ \frac{x}{4} \cdot 4 &= 2 \cdot 4 \\ x &= 8 \end{aligned}$$

(e) $2 \log_2 x - \log_2 3 = 2$

$$\begin{aligned} 2 \log_2 x - \log_2 3 &= 2 \\ \log_2 x^2 - \log_2 3 &= 2 \\ \log_2 \frac{x^2}{3} &= 2 \\ \frac{x^2}{3} &= 2^2 \\ \frac{x^2}{3} \cdot 3 &= 2^2 \cdot 3 \\ x^2 &= 12 \\ x &= \pm\sqrt{12} = \pm 2\sqrt{3} \end{aligned}$$

However, x must be positive, since the domain of $\log_2 x$ is positive numbers only. Hence the solution is $x = 2\sqrt{3}$.

(f) $\log_5(2x - 3) + \log_5 x = 1$

$$\begin{aligned} \log_5(2x - 3) + \log_5 x &= 1 \\ \log_5[(2x - 3)x] &= 1 \\ (2x - 3)x &= 5^1 \\ 2x^2 - 3x &= 5 \\ 2x^2 - 3x - 5 &= 5 - 5 \\ 2x^2 - 3x - 5 &= 0 \\ (2x - 5)(x + 1) &= 0 \\ 2x - 5 = 0 \quad \text{or} \quad x + 1 = 0 \\ 2x = 5 \quad \quad x = -1 \\ x = \frac{5}{2} \end{aligned}$$

$x = -1$ is extraneous, since it would give us a log of a negative number. The only solution is $x = \frac{5}{2}$.

36. The half-life of some naturally occurring radioactive isotope is 200 years. How long would it take for a sample of this isotope to decay to one-tenth of the initial amount?

We have $A(t) = A_0 e^{-kt}$. In 200 years, the amount left will be $A_0/2$. Then

$$\begin{aligned} \frac{A_0}{2} &= A_0 e^{-k(200)} \\ \frac{1}{A_0} \cdot \frac{A_0}{2} &= \frac{1}{A_0} \cdot A_0 e^{-200k} \\ \frac{1}{2} &= e^{-200k} \\ \ln \frac{1}{2} &= \ln e^{-200k} \\ \ln \frac{1}{2} &= -200k \ln e \\ \ln \frac{1}{2} &= -200k \\ k &= \frac{\ln \frac{1}{2}}{-200} \\ k &\approx 0.003465736 \end{aligned}$$

We are looking for the value of t for which $A(t) = \frac{A_0}{10}$.

$$\begin{aligned} \frac{A_0}{10} &= A_0 e^{-0.003465736t} \\ \frac{1}{10} &= e^{-0.003465736t} \end{aligned}$$

$$\begin{aligned}
\ln \frac{1}{10} &= \ln e^{-0.003465736t} \\
-0.003465736t &= \ln \frac{1}{10} \\
\frac{-0.003465736t}{-0.003465736} &= \frac{\ln \frac{1}{10}}{-0.003465736} \\
t &= \frac{\ln \frac{1}{10}}{-0.003465736} \\
t &\approx 664 \text{ years}
\end{aligned}$$

37. Solve the systems of equations on paper.

(a)

$$\begin{aligned}
2x + y &= 2 \\
x + 2y &= -5
\end{aligned}$$

Multiply the second equation by -2 and add it to the first.

$$\begin{array}{r}
2x + y = 2 \\
-2x - 4y = 10 \\
\hline
-3y = 12
\end{array}$$

$$\begin{aligned}
\frac{-3y}{-3} &= \frac{12}{-3} \\
y &= -4
\end{aligned}$$

We use this value along with the first equation to determine x .

$$\begin{aligned}
2x + (-4) &= 2 \\
2x - 4 + 4 &= 2 + 4 \\
2x &= 6 \\
\frac{2x}{2} &= \frac{6}{2} \\
x &= 3
\end{aligned}$$

The solution is the point $(3, -4)$.

(b)

$$\begin{aligned}
2x - y &= 9 \\
y &= 3x - 11
\end{aligned}$$

We can use the substitution method, using $3x - 11$ in place of y in the first equation.

$$\begin{aligned}
2x - (3x - 11) &= 9 \\
2x - 3x + 11 &= 9 \\
-x + 11 - 11 &= 9 - 11 \\
-x &= -2 \\
x &= 2
\end{aligned}$$

Using this value in the second equation, we have

$$y = 3(2) - 11 = 6 - 11 = -5$$

The solution is $(2, -5)$.

(c)

$$\begin{aligned} 2x + y - z &= 0 \\ x + 2y + z &= 3 \\ 4x - y + z &= 18 \end{aligned}$$

We can add the first and second equations together to eliminate z . We can also add the first and third together to eliminate z .

$$\begin{array}{r} 2x + y - z = 0 \\ x + 2y + z = 3 \\ \hline 3x + 3y = 3 \end{array}$$

$$\begin{array}{r} 2x + y - z = 0 \\ 4x - y + z = 18 \\ \hline 6x = 18 \end{array}$$

We have gotten a bonus with the second pair, in that y went away along with x . Solve the remaining equation for x .

$$\begin{aligned} \frac{6x}{6} &= \frac{18}{6} \\ x &= 3 \end{aligned}$$

Using this value in the equation we got from the first pair to determine y ,

$$\begin{aligned} 3(3) + 3y &= 3 \\ 9 + 3y - 9 &= 3 - 9 \\ 3y &= -6 \\ \frac{3y}{3} &= \frac{-6}{3} \\ y &= -2 \end{aligned}$$

We can use these values for x and y in the first equation to determine z .

$$\begin{aligned} 2(3) + (-2) - z &= 0 \\ 4 - z &= 0 \\ 4 - z + z &= 0 + z \\ z &= 4 \end{aligned}$$

The solution is $(3, -2, 4)$.

38. Solve the systems using your choice of method, including matrix methods on the calculator (recommended).

We are using the method outlined in the supplement Solving systems of linear equations on the TI-83. Other methods can be used. If you use this method, write down the augmented matrix you enter into your calculator to solve. In general, tell me what you are doing.

(a)

$$\begin{aligned}45x+18y+12z&=69 \\63x+27y+15z&=92 \\18x+9y+6z&=32\end{aligned}$$

The augmented coefficient matrix is

$$\left[\begin{array}{cccc} 45 & 18 & 12 & 69 \\ 63 & 27 & 15 & 92 \\ 18 & 9 & 6 & 32 \end{array} \right]$$

The matrix we get, after converting to fractions, is

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 5/9 \\ 0 & 1 & 0 & 4/9 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Translating this to a system of equations, this is

$$\begin{aligned}x &= \frac{5}{9} \\y &= \frac{4}{9} \\z &= 3\end{aligned}$$

(b)

$$\begin{aligned}4x \quad +10z &= 21 \\3y - 5z &= -10 \\8x+6y+15z &= 30\end{aligned}$$

The augmented coefficient matrix is

$$\left[\begin{array}{cccc} 4 & 0 & 10 & 21 \\ 0 & 3 & -5 & -10 \\ 8 & 6 & 15 & 30 \end{array} \right]$$

The matrix we get, after converting to fractions, is

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 5/4 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 8/5 \end{array} \right]$$

Translating this to a system of equations, this is

$$\begin{aligned}x &= \frac{5}{4} \\y &= -\frac{2}{3} \\z &= \frac{8}{5}\end{aligned}$$

(c)

$$\begin{aligned}24x+18y+30z&=71 \\8x+ 6y+10z&=24 \\12x+ 6y+20z&=45\end{aligned}$$

The augmented coefficient matrix is

$$\begin{bmatrix} 24 & 18 & 30 & 71 \\ 8 & 6 & 10 & 24 \\ 12 & 6 & 20 & 45 \end{bmatrix}$$

The matrix we get, after converting to fractions, is

$$\begin{bmatrix} 1 & 0 & 5/2 & 0 \\ 0 & 1 & -5/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translating this to a system of equations, this is

$$\begin{aligned}x + \frac{5}{2}z &= 0 \\y - \frac{5}{3}z &= 0 \\0 &= 1\end{aligned}$$

Since the last equation can never be true, there is no solution. The system is inconsistent.