Trigonometry Exam 1B Solutions
Given on February 9, 2009

All nontrivial calculations must be shown.

1. For the equation $5x^2 - y = 10$, find the x-intercepts and y-intercepts and test for symmetry about the x-axis, y-axis, and origin.

   - x-intercepts: Let $y = 0$ and solve for x.
   $$\begin{aligned} 5x^2 - (0) &= 10 \\ \frac{5x^2}{5} &= \frac{10}{5} \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$
   - y-intercept: Let $x = 0$ and solve for y.
   $$\begin{aligned} 5(0)^2 - y &= 10 \\ -y &= 10 \\ y &= -10 \end{aligned}$$
   - Symmetry about x-axis: Replace y with $-y$ and simplify.
   $$\begin{aligned} 5x^2 - (-y) &= 10 \\ 5x^2 + y &= 10 \end{aligned}$$
   Since this is not equivalent to the original equation, its graph is not symmetric with respect to the x-axis.
   - Symmetry about y-axis: Replace x with $-x$ and simplify.
   $$\begin{aligned} 5(-x)^2 - y &= 10 \\ 5x^2 - y &= 10 \end{aligned}$$
   Since this is equivalent to the original equation, its graph is symmetric with respect to the y-axis.
   - Symmetry about origin: We could check this by replacing x with $-x$ and y with $-y$, but it is not really necessary. It is not possible for a graph to exhibit exactly two of the three types of symmetry, hence the graph cannot be symmetric about the origin.

2. State the center and radius of the circle
$$(x-4)^2 + (y+3)^2 = 16$$

   Comparing this to the general form $(x-h)^2 + (y-k)^2 = R^2$, we have
   $$h = 4 \qquad k = -3 \qquad \text{and} \qquad R = \sqrt{16} = 4$$
   The circle has its center at $(4, -3)$ and has radius 4.

3. Find the domains of the functions.

(a) $f(x) = \sqrt{12 - 3x}$

Even roots of negative numbers are not Real numbers, hence we must insure that the radicand is not negative.

$$\begin{aligned} 12 - 3x &\geq 0 \\ 12 - 3x + 3x &\geq 0 + 3x \\ 12 &\geq 3x \\ \frac{12}{3} &\geq \frac{3x}{3} \\ 4 &\geq x \end{aligned}$$

The domain is $\{x|x \leq 4\}$.

(b) $g(x) = \dfrac{x^2 + 5}{x^2 - 3x}$

Division by zero is not defined, hence we must insure that the denominator is not zero. We set the denominator equal to zero and solve.

$$\begin{aligned} x^2 - 3x &= 0 \\ x(x-3) &= 0 \\ x = 0 \quad &\text{or} \quad x - 3 = 0 \\ & \quad\quad x = 3 \end{aligned}$$

So 0 and 3 are numbers that *cannot* be included in the domain. The domain is $\{x|x \neq 0, 3\}$.

4. Use the graph of the function to find

(a) The domain: $[-9, 1] \cup [3, 7]$

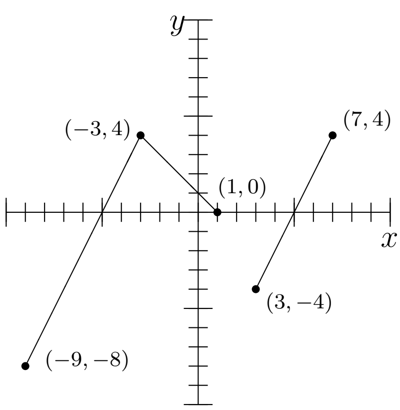
(b) The range: $[-8, 4]$

(c) y-intercept: $(0, 1)$

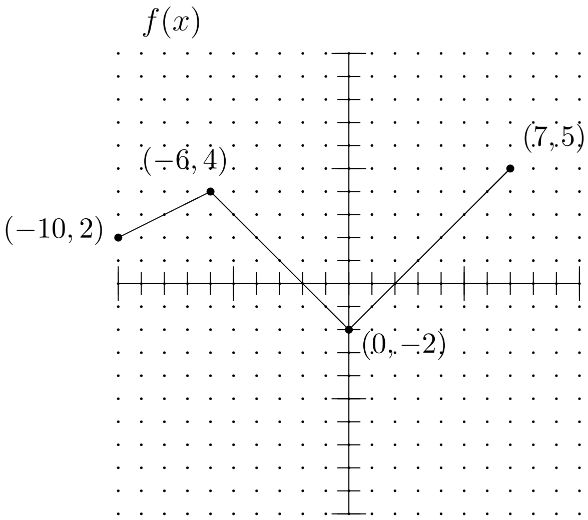
(d) x-intercept(s): $(-5, 0), (1, 0), (5, 0)$

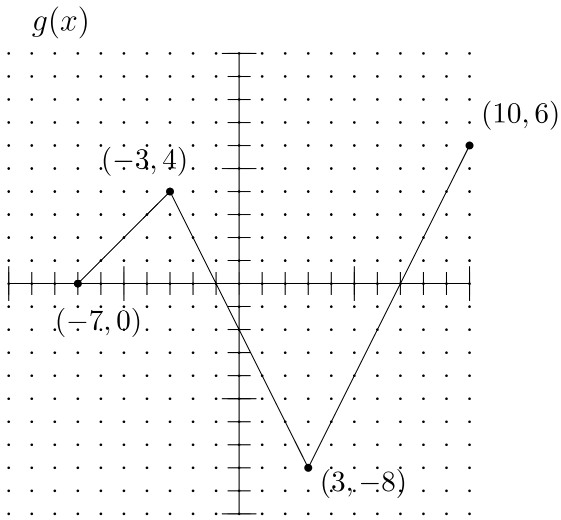
(e) Increasing intervals: $(-9, -3) \cup (3, 7)$

(f) Decreasing intervals: $(-3, 1)$



5. Shown below is a function $f(x)$. Graph the related function $g(x) = 2f(x-3) - 4$





The function $g(x)$ is $f(x)$ shifted to the right by 3 units, stretched along the y-axis by a factor of 2, and shifted down the y-axis by 4 units, in that order. We first determine where the labeled points will end up.

point	right by 3	y-stretch by 2	down by 4
$(-10, 2)$	$(-7, 2)$	$(-7, 4)$	$(-7, 0)$
$(-6, 4)$	$(-3, 4)$	$(-3, 8)$	$(-3, 4)$
$(0, -2)$	$(3, -2)$	$(3, -4)$	$(3, -8)$
$(7, 5)$	$(10, 5)$	$(10, 10)$	$(10, 6)$

Now we connect those points with line segments, and we have the transformed graph.

6. Convert the angles from degrees to radians.

   (a) $210° = \underline{\frac{7\pi}{6}}$

$$210° \times \frac{\pi}{180°} = \frac{30(7)\pi}{30(6)} = \frac{7\pi}{6}$$

   (b) $120° = \underline{\frac{2\pi}{3}}$

$$120° \times \frac{\pi}{180°} = \frac{60(2)\pi}{60(3)} = \frac{2\pi}{3}$$

7. Convert the angles from radians to degrees.

   (a) $\frac{5\pi}{3} = \underline{300°}$

$$\frac{5\pi}{3} \times \frac{180°}{\pi} = \frac{5(3)(60°)(\pi)}{3(\pi)} = 5(60°) = 300°$$

(b) $1.3 = \underline{\ 74.48^\circ\ }$

$$1.3 \times \frac{180^\circ}{\pi} = \frac{234^\circ}{\pi} \approx 74.48^\circ$$

8. Find the missing quantity.

(a) $r = 5$" $s = 7.5$" $\theta = \underline{\ 1.5\text{rad}\ }$

$$\theta = \frac{s}{r} = \frac{7.5"}{5"} = 1.5\text{rad}$$

(b) $\theta = 1.9$ $r = 6$ meters $s = \underline{\ 11.4 \text{ meters}\ }$

$$s = r\theta = 6(1.9) = 11.4 \text{ meters}$$

(c) $\theta = 25^\circ$ $r = 4$ ft $s = \underline{\ 1.75\text{ft}\ }$

$$\theta = 25^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{36}$$

$$s = r\theta = 4 \times \frac{5\pi}{36} = \frac{20\pi}{36} = \frac{5\pi}{9} \approx 1.75\text{ft}$$

Note that θ must be in radians

9. Express the angle $21^\circ 42' 15''$ in decimal degrees and then again in radians.

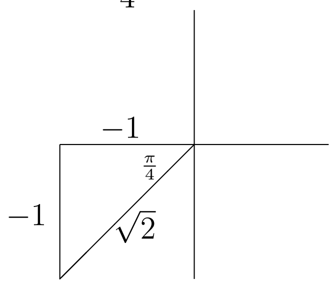
$$21^\circ 42' 15'' = 21^\circ + \frac{42^\circ}{60} + \frac{15^\circ}{3600} \approx 21.7042^\circ$$

In radians, this is

$$21.7042^\circ \times \frac{\pi}{180^\circ} = .37881\text{rad}$$

10. Find the *exact* values (not the calculator approximations) of the following trigonometric values.

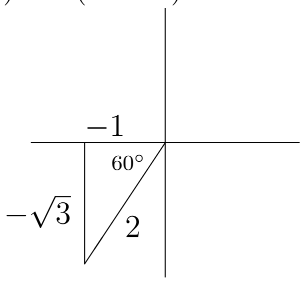
(a) $\cos \frac{5\pi}{4}$



$\frac{5\pi}{4} = \pi + \frac{\pi}{4}$, so the angle is in quadrant III and the reference angle is $\frac{\pi}{4}$. This is the standard 45° triangle. In quadrant III, x and y are both negative. Then

$$\cos \frac{5\pi}{4} = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

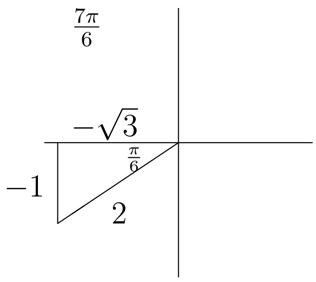
(b) $\sin(-120°)$

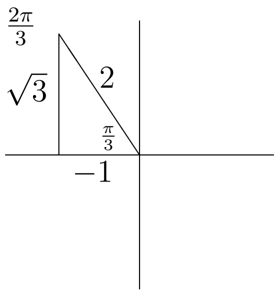


Negative angles are measure in the clockwise direction from the positive x-axis. $120° = 180° - 60°$, so this angle is in quadrant III, and the reference angle is $60°$. This is the standard $30° - 60° - 90°$ triangle. Again in quadrant III, x and y are both negative. Then

$$\sin(-120°) = \frac{\text{opp}}{\text{hyp}} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

(c) $\cot\frac{7\pi}{6} - \csc\frac{2\pi}{3}$





$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$, so $\frac{7\pi}{6}$ is in quadrant III, the reference angle is $\frac{\pi}{6}$ and

$$\cot\frac{7\pi}{6} = \frac{\text{adj}}{\text{opp}} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

$\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, so $\frac{2\pi}{3}$ is in quadrant II, the reference angle is $\frac{\pi}{3}$, and

$$\csc\frac{2\pi}{3} = \frac{\text{hyp}}{\text{opp}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Then

$$\cot\frac{7\pi}{6} - \csc\frac{2\pi}{3} = \sqrt{3} - \frac{2\sqrt{3}}{3} = \frac{3\sqrt{3}}{3} - \frac{2\sqrt{3}}{3} = \frac{\sqrt{3}}{3}$$

(d) $\cos 10° - \frac{\cot 10°}{\csc 10°}$

$$\cos 10° - \frac{\cot 10°}{\csc 10°} = \cos 10° - \frac{\cos 10°}{\sin 10°} \div \frac{1}{\sin 10°}$$

$$\begin{aligned} &= \cos 10^\circ - \frac{\cos 10^\circ}{\sin 10^\circ} \cdot \frac{\sin 10^\circ}{1} \\ &= \cos 10^\circ - \cos 10^\circ \\ &= 0 \end{aligned}$$

11. Use a calculator to find approximations for the following trigonometric values.

 (a) $\sin 80^\circ =$ _0.9848_

 Make sure your calculator is in *degree* mode.

 (b) $\csc \frac{3\pi}{7} =$ _1.0257_

 Make sure your calculator is in *radian* mode.

$$\csc \frac{3\pi}{7} = \frac{1}{\sin \frac{3\pi}{7}} \approx 1.0257$$

 (c) $\sec(-1.5) =$ _14.1368_

 Make sure your calculator is in *radian* mode.

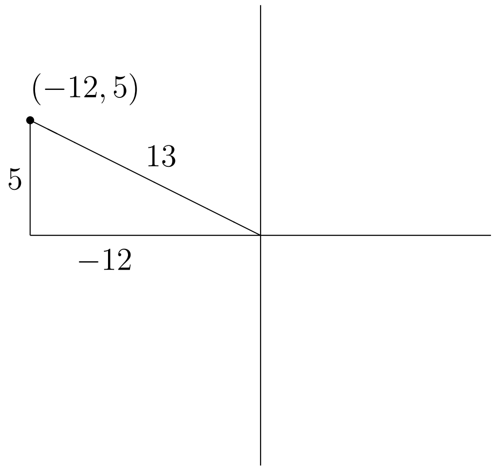
$$\sec(-1.5) = \frac{1}{\cos(-1.5)} \approx 14.1368$$

12. The point (-12,5) is on the terminal side of an angle in standard position. Find the values of the trigonometric functions for this angle.

$\sin\theta =$ _$\frac{5}{13}$_ $\csc\theta =$ _$\frac{13}{5}$_

$\cos\theta =$ _$-\frac{12}{13}$_ $\sec\theta =$ _$-\frac{13}{12}$_

$\tan\theta =$ _$-\frac{5}{12}$_ $\cot\theta =$ _$-\frac{12}{5}$_



We can calculate the hypotenuse using the Pythagorean theorem.

$$
\begin{aligned}
h^2 &= 5^2 + 12^2 \\
&= 25 + 144 \\
&= 169 \\
h &= \sqrt{169} \\
&= 13
\end{aligned}
$$