MATH 128 Exam # 2B Solutions
Given on March 9, 2009

Show all nontrivial calculations.

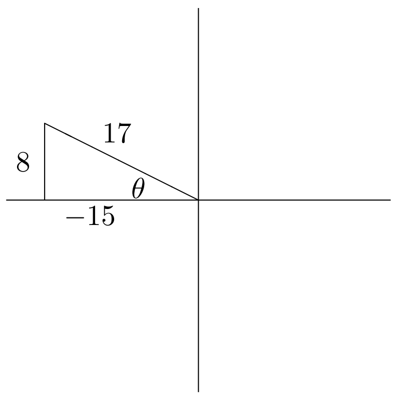
1. If $\sin\theta = \frac{8}{17}$ and $\tan\theta < 0$, find the values of all trigonometric functions.

$\sin\theta = \underline{\frac{8}{17}}$ $\csc\theta = \underline{\frac{17}{8}}$

$\cos\theta = \underline{-\frac{15}{17}}$ $\sec\theta = \underline{-\frac{17}{15}}$

$\tan\theta = \underline{-\frac{8}{15}}$ $\cot\theta = \underline{-\frac{15}{8}}$

Since $\sin\theta$ is positive, θ is in either quadrant I or quadrant II. Since $\tan\theta$ is negative, it must be quadrant II. This gives us the picture below.



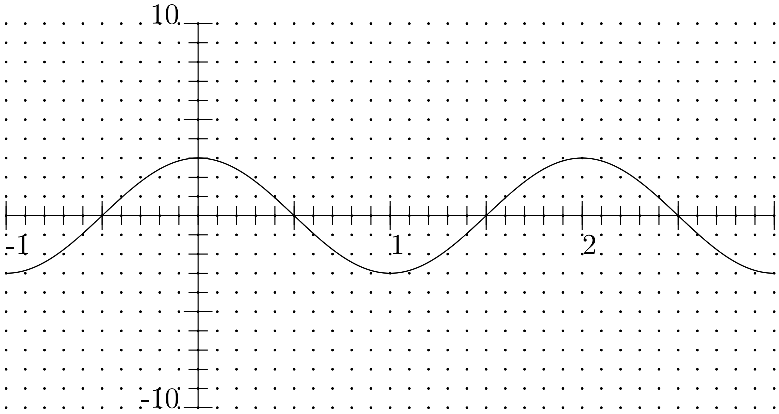
The base of the triangle is obtained from the Pythagorean theorem.

$$\begin{aligned} 17^2 &= 8^2 + x^2 \\ x^2 &= 17^2 - 8^2 \\ x &= \pm\sqrt{17^2 - 8^2} \\ x &= \pm 15 \end{aligned}$$

Since the angle is in quadrant II, $x = -15$.

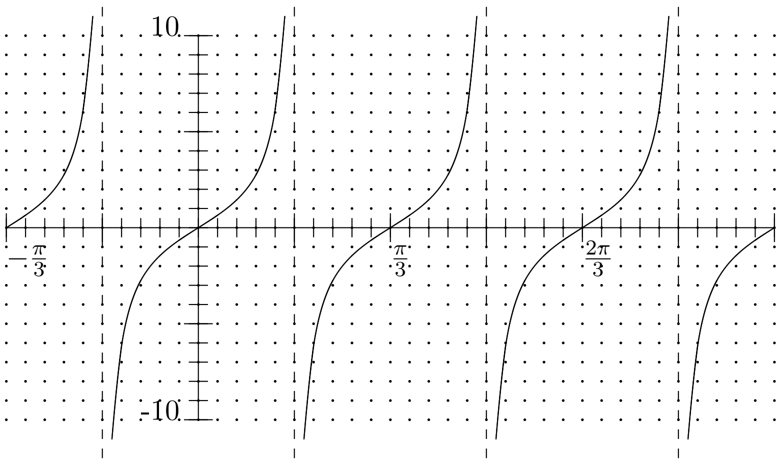
2. Graph at least one period of the following functions. *Number the axes.* Also find the amplitude (where applicable), the period and the phase shift.

   (a) $f(x) = 3\cos(\pi x)$



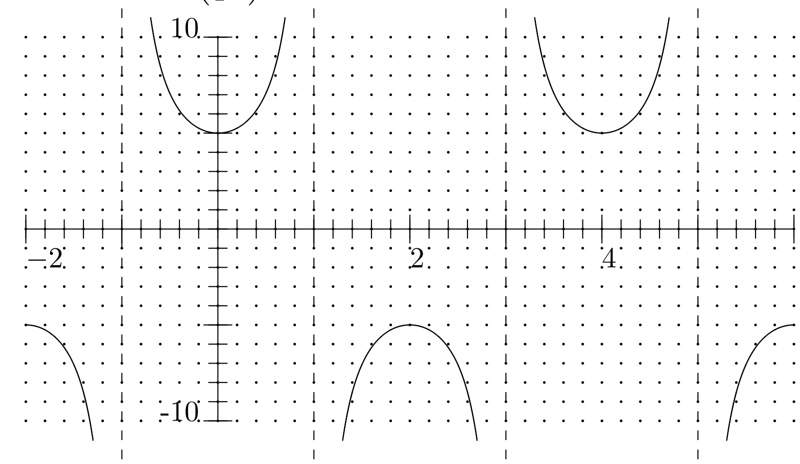
Amplitude 3 Period $\frac{2\pi}{\pi} = 2$ Phase Shift none

   (b) $g(x) = 2\tan(3x)$



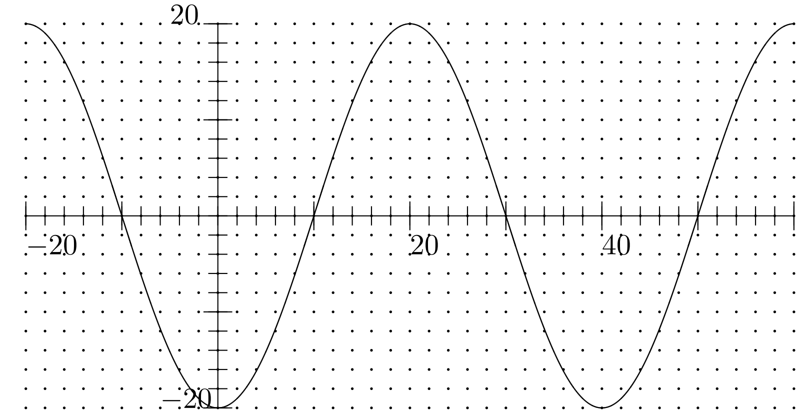
Amplitude none Period $\frac{\pi}{3}$ Phase Shift none

(c) $h(x) = 5\sec\left(\frac{\pi}{2}x\right)$



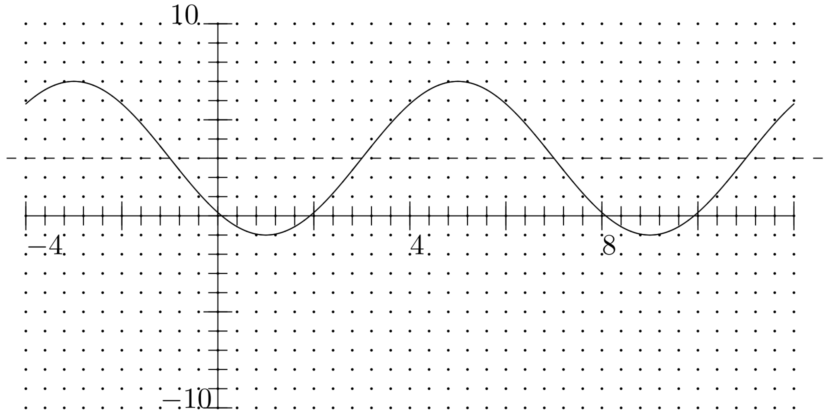
Amplitude <u>none</u> Period <u>$\frac{2\pi}{\pi/2} = 4$</u> Phase Shift <u>none</u>

(d) $f(x) = 20\sin\left(\frac{\pi}{20}x - \frac{\pi}{2}\right)$



Amplitude <u>20</u> Period <u>$\frac{2\pi}{\pi/20} = 40$</u> Phase Shift <u>$\frac{\pi/2}{\pi/20} = 10$</u>

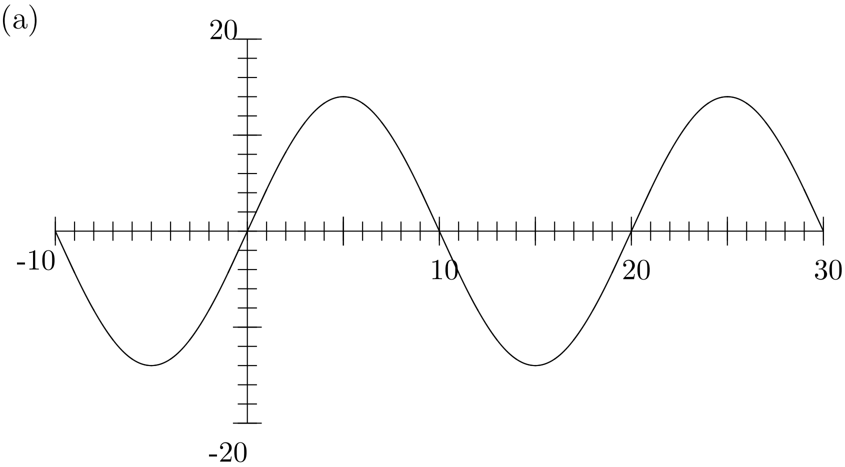
(e) $g(x) = -4\cos(\frac{\pi}{4}x - \frac{\pi}{4}) + 3$



Amplitude <u>$|-4| = 4$</u> Period <u>$\frac{2\pi}{\pi/4} = 8$</u> Phase Shift <u>$\frac{\pi/4}{\pi/4} = 1$</u>

3. Find equations for the functions graphed.

(a)



By examination, we see that one cycle of a standard sine function begins at zero on the graph. So we can use a sine function with no phase shift. The form of the equation is $y = A \sin \omega x + B$. the maximum y-value is 14 and the minimum y-value is -14, so

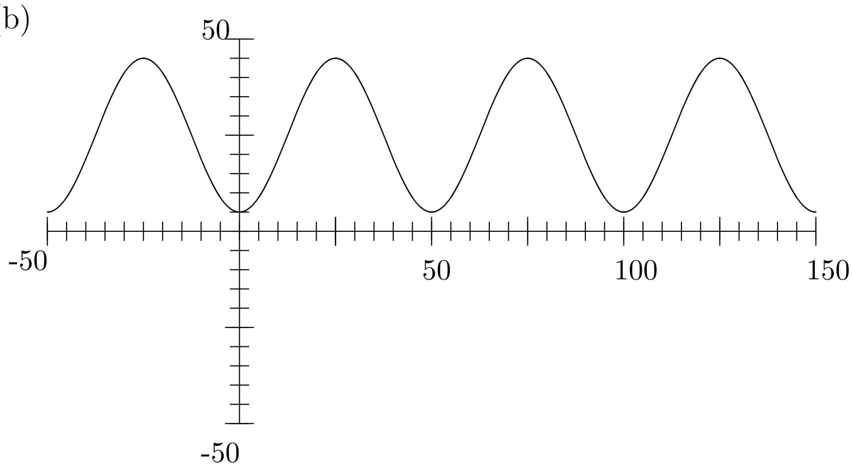
$$A = \frac{\max - \min}{2} = \frac{14 - (-14)}{2} = 14 \qquad B = \frac{\max + \min}{2} = \frac{14 + (-14)}{2} = 0$$

to determine ω, we note that one complete cycle starts at zero and ends at 20, so the period is $T = 20 - 0 = 20$. Then

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ \omega T &= 2\pi \\ \omega &= \frac{2\pi}{T} \\ \omega &= \frac{2\pi}{20} \\ \omega &= \frac{\pi}{10} \end{aligned}$$

Then one equation of the function is $y = 14 \sin\left(\frac{\pi}{10}x\right)$. Note that this is not the only way to represent this function. The same function could be rendered as $y = 14 \cos\left(\frac{\pi}{10}x - \frac{\pi}{2}\right)$, for example.

(b)



Since the function has a minimum at zero on the x-axis, we can represent it as $y = -A\cos(\omega x) + B$ with no phase shift. The maximum y-value is 45 and the minimum y-value is 5.

$$A = \frac{\max - \min}{2} = \frac{45 - (5)}{2} = 20 \qquad B = \frac{\max + \min}{2} = \frac{45 + (5)}{2} = 25$$

One complete cycle occurs between 0 and 50 on the x-axis, so the period is $T = 50 - 0 = 50$.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{50} = \frac{\pi}{25}$$

so one equation of the function is $y = -20\cos\left(\frac{\pi}{25}x\right) + 25$. Again, this is not unique. Another way of expressing the same function is $y = 20\cos\left(\frac{\pi}{25}x - \pi\right) + 25$.

4. Show that the functions $f(x) = \dfrac{3x-2}{x}$ and $g(x) = \dfrac{2}{3-x}$ are inverse functions.

Graphing the functions shows that they are one-to-one functions, since they satisfy both horizontal and vertical line tests.

$$\begin{aligned} f(g(x)) &= f\left(\frac{2}{3-x}\right) \\ &= \frac{3\left(\frac{2}{3-x}\right) - 2}{\left(\frac{2}{3-x}\right)} \\ &= \frac{\frac{6}{3-x} - 2}{\frac{2}{3-x}} \\ &= \frac{\frac{6}{3-x} - 2}{\frac{2}{3-x}} \cdot \frac{3-x}{3-x} \\ &= \frac{6 - 2(3-x)}{2} \end{aligned}$$

$$
\begin{aligned}
&= \frac{6-6+2x}{2} \\
&= \frac{2x}{2} \\
&= x
\end{aligned}
$$

$$
\begin{aligned}
g\left(f(x)\right) &= g\left(\frac{3x-2}{x}\right) \\
&= \frac{2}{3-\left(\frac{3x-2}{x}\right)} \\
&= \frac{2}{3-\left(\frac{3x-2}{x}\right)} \cdot \frac{x}{x} \\
&= \frac{2x}{3x-(3x-2)} \\
&= \frac{2x}{3x-3x+2} \\
&= \frac{2x}{2} \\
&= x
\end{aligned}
$$

5. Find inverse functions for the following.

   (a) $f(x) = 5x + 3$

$$
\begin{aligned}
y &= 5x+3 \\
y-3 &= 5x \\
\frac{5x}{5} &= \frac{y-3}{5} \\
x &= \frac{y-3}{5} \\
f^{-1}(x) &= \frac{x-3}{5}
\end{aligned}
$$

   (b) $g(x) = \dfrac{x}{x+1}$

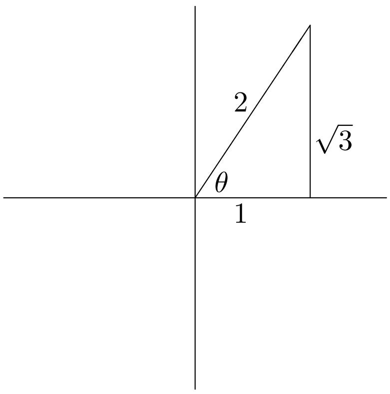
$$
\begin{aligned}
y &= \frac{x}{x+1} \\
y \cdot (x+1) &= \frac{x}{x+1} \cdot (x+1) \\
yx + y &= x
\end{aligned}
$$

$$
\begin{aligned}
x - yx &= y \\
x(1-y) &= y \\
\frac{x(1-y)}{1-y} &= \frac{y}{1-y} \\
x &= \frac{y}{1-y} \\
f^{-1}(x) &= \frac{x}{1-x}
\end{aligned}
$$

6. Find exact values in radians for the following expressions.

   (a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

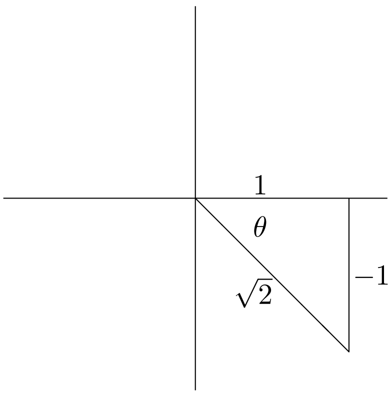
   $\sin^{-1} x$ gives us an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Since $\frac{\sqrt{3}}{2}$ is positive, the angle must be in quadrant I. this gives us the picture below.

   These are the dimensions of the standard 30-60-90 triangle, so $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60° = \frac{\pi}{3}$.

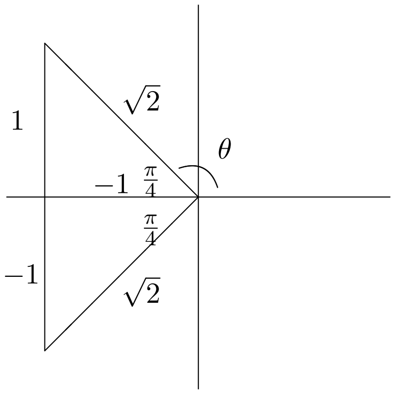
   (b) $\tan^{-1}(-1)$

   $\tan^{-1} x$ gives us an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Since -1 is negative, the angle must be in quadrant IV. this gives us the picture below.

   These are the dimensions of the standard 45° right triangle, so $\tan^{-1}(-1) = -\frac{\pi}{4}$.

(c) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$



$\cos^{-1} x$ must be an angle between 0 and π. We are looking for an angle between 0 and π which has the same cosine as the angle $\frac{5\pi}{4}$. Since $\cos\frac{5\pi}{4}$ is negative, the angle we are looking for must be in quadrant II. Since this angle must have the same reference angle as $\frac{5\pi}{4}$, the angle must be

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

7. Use your calculator to evaluate the following in degrees.

(a) $\cos^{-1}(.65) \approx$ __49.46°____

(b) $\tan^{-1}(1.25) \approx$ __51.34∘____