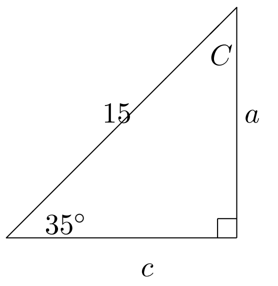
MATH 128 Exam #3 Solutions
Given on April 6, 2009

Show all non-trivial calculations. Triangles are not drawn to scale.

1. Find the remaining angle and sides of the right triangle.

(a)

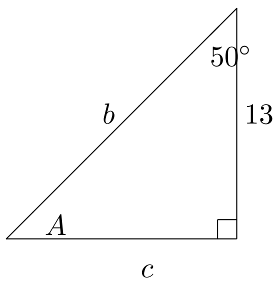


$$C = 180° - 90° - 35° = 55°$$

$$\begin{aligned} \sin 35° &= \frac{a}{15} \\ \sin 35° \cdot 15 &= \frac{a}{15} \cdot 15 \\ a &= 15 \sin 35° \\ a &= 8.60 \end{aligned}$$

$$\begin{aligned} 15^2 &= 8.60^2 + c^2 \\ c^2 &= 15^2 - 8.60^2 \\ c &= \sqrt{15^2 - 8.60^2} \\ c &= 12.29 \end{aligned}$$

(b)

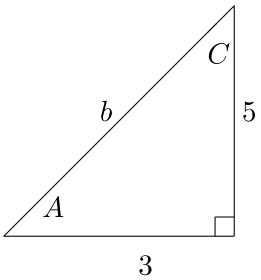


$$A = 180° - 90° - 50° = 40°$$

$$\begin{aligned} \cos 50° &= \frac{13}{b} \\ b \times \cos 50° &= b \times \frac{13}{b} \end{aligned}$$

$$
\begin{aligned}
b\cos 50^\circ &= 13 \\
\frac{b\cos 50^\circ}{\cos 50^\circ} &= \frac{13}{\cos 50^\circ} \\
b &= \frac{13}{\cos 50^\circ} \\
b &= 20.22
\end{aligned}
$$

$$
\begin{aligned}
20.22^2 &= c^2 + 13^2 \\
20.22^2 - 13^2 &= c^2 + 13^2 - 13^2 \\
c &= \sqrt{20.22^2 - 13^2} \\
c &= 15.49
\end{aligned}
$$

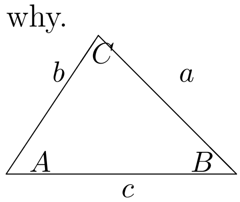
(c) 

$$
\begin{aligned}
b^2 &= 3^2 + 5^2 \\
B &= \sqrt{3^2 + 5^2} \\
b &= 5.83
\end{aligned}
$$

$$
\begin{aligned}
\tan A &= \frac{5}{3} \\
A &= \tan^{-1}\left(\frac{5}{3}\right) \\
A &= 59.04^\circ
\end{aligned}
$$

$$C = 180^\circ - 90^\circ - 59.04^\circ = 30.96^\circ$$

2. Find the remaining angles and sides of the oblique triangles. If no triangle is possible, explain why.



(a)

$A = 45^\circ$	$B =$ ____	$C = 65^\circ$
$a = 17$	$b =$ ____	$c =$ ____

$$\begin{aligned}
\frac{\sin 45^\circ}{17} &= \frac{\sin 65^\circ}{c} \\
c \sin 45^\circ &= 17 \sin 65^\circ \\
c &= 17 \frac{\sin 65^\circ}{\sin 45^\circ} \\
c &= 21.79
\end{aligned}$$

$$B = 180^\circ - 45^\circ - 65^\circ = 70^\circ$$

$$\begin{aligned}
\frac{\sin 45^\circ}{17} &= \frac{\sin 70^\circ}{b} \\
b \sin 45^\circ &= 17 \sin 70^\circ \\
b &= 17 \frac{\sin 70^\circ}{\sin 45^\circ} \\
b &= 22.59
\end{aligned}$$

(b)

$$\begin{array}{ccc}
A = 50^\circ & B = 80^\circ & C = \underline{\qquad} \\
a = \underline{\qquad} & b = \underline{\qquad} & c = 18
\end{array}$$

$$C = 180^\circ - 50^\circ - 80^\circ = 50^\circ$$

Since $C = A$, the triangle is isosceles, and $a = c = 18$

$$\begin{aligned}
\frac{\sin 50^\circ}{18} &= \frac{\sin 80^\circ}{b} \\
b \sin 50^\circ &= 18 \sin 80^\circ \\
b &= 18 \frac{\sin 80^\circ}{\sin 50^\circ} \\
b &= 23.14
\end{aligned}$$

(c)

$$\begin{array}{ccc}
A = 55^\circ & B = \underline{\qquad} & C = \underline{\qquad} \\
a = 12 & b = 7 & c = \underline{\qquad}
\end{array}$$

This is the SSA case, but since side a opposite the known angle is the larger of the two known sides, there is one unique triangle.

$$\begin{aligned}
\frac{\sin 55^\circ}{12} &= \frac{\sin B}{7} \\
7 \times \frac{\sin 55^\circ}{12} &= 7 \times \frac{\sin B}{7} \\
\sin B &= 7 \times \frac{\sin 55^\circ}{12} \\
B &= \sin^{-1}\left(7 \times \frac{\sin 55^\circ}{12}\right) \\
B &= 28.54^\circ
\end{aligned}$$

$$C = 180° - 55° - 28.54° = 96.46°$$

$$\begin{aligned}
\frac{\sin 55°}{12} &= \frac{\sin 96.46°}{c} \\
c \sin 55° &= 12 \sin 96.46° \\
c &= 12 \frac{\sin 96.46°}{\sin 55°} \\
c &= 14.56
\end{aligned}$$

(d)

$$\begin{array}{lll}
A = 60° & B = ____ & C = ____ \\
a = 17 & b = 18 & c = ____
\end{array}$$

This again is the SSA case, but this time the side opposite the known angle is the shorter side. We expect to find two triangles.

$$\begin{aligned}
\frac{\sin 60°}{17} &= \frac{\sin B}{18} \\
\sin B &= 18 \frac{\sin 60°}{17} \\
B &= \sin^{-1}\left(18 \frac{\sin 60°}{17}\right) \\
B &= 66.49°
\end{aligned}$$

But there is also a B', given by

$$B' = 180° - B = 180° - 66.49° = 113.51°$$

$$C = 180° - 60° - 66.49° = 53.51°$$

and

$$C' = 180° - 60° - 113.51° = 6.49°$$

$$\begin{aligned}
\frac{\sin 60°}{17} &= \frac{\sin 53.51°}{c} \\
c \sin 60° &= 17 \sin 53.51° \\
c &= 17 \frac{\sin 53.51°}{\sin 60°} \\
c &= 15.78
\end{aligned}$$

$$\begin{aligned}
\frac{\sin 60°}{17} &= \frac{\sin 6.49°}{c'} \\
c' \sin 60° &= 17 \sin 6.49° \\
c' &= 17 \frac{\sin 6.49°}{\sin 60°} \\
c' &= 2.22
\end{aligned}$$

(e)

$$\begin{array}{ccc} A = 65 & B = \_\_\_\_ & C = \_\_\_\_ \\ a = 12 & b = 18 & c = \_\_\_\_ \end{array}$$

Again the side opposite the known angle is the shorter side. We expect to find two triangles.

$$\begin{aligned} \frac{\sin 65°}{12} &= \frac{\sin B}{18} \\ \sin B &= 18\frac{\sin 65°}{12} \\ B &= \sin^{-1}\left(18\frac{\sin 65°}{12}\right) \end{aligned}$$

When we attempt to evaluate this on the calculator, we get a domain error. In fact

$$18\frac{\sin 65°}{12} = 1.36$$

Since this is greater than one the triangle is not possible.

(f)

$$\begin{array}{ccc} A = \_\_\_\_ & B = 47° & C = \_\_\_\_ \\ a = 25 & b = \_\_\_\_ & c = 17 \end{array}$$

We can use the law of cosines to find side b.

$$\begin{aligned} b^2 &= 25^2 + 17^2 - 2(25)(17)\cos 47° \\ b &= \sqrt{25^2 + 17^2 - 2(25)(17)\cos 47°} \\ b &= 18.28 \end{aligned}$$

We can also use the law of cosines to find angle A.

$$\begin{aligned} 25^2 &= 17^2 + 18.28^2 - 2(17)(18.28)\cos A \\ 25^2 - 17^2 - 18.28^2 &= 17^2 + 18.28^2 - 2(17)(18.28)\cos A - 7^2 - 18.28^2 \\ 25^2 - 17^2 - 18.28^2 &= -2(17)(18.28)\cos A \\ \frac{-2(17)(18.28)\cos A}{-2(17)(18.28)} &= \frac{25^2 - 17^2 - 18.28^2}{-2(17)(18.28)} \\ \cos A &= \frac{25^2 - 17^2 - 18.28^2}{-2(17)(18.28)} \\ A &= \cos^{-1}\left(\frac{25^2 - 17^2 - 18.28^2}{-2(17)(18.28)}\right) \\ A &= 90.17° \end{aligned}$$

$$C = 180° - 47° - 90.17° = 42.83°$$

(g)

$$A = \_\_\_\_ \qquad B = \_\_\_\_ \qquad C = \_\_\_\_$$
$$a = 15 \qquad b = 17 \qquad c = 9$$

$$\begin{aligned}
17^2 &= 15^2 + 9^2 - 2(15)(9)\cos B \\
17^2 - 15^2 - 9^2 &= 15^2 + 9^2 - 2(15)(9)\cos B - 15^2 - 9^2 \\
17^2 - 15^2 - 9^2 &= -2(15)(9)\cos B \\
\frac{-2(15)(9)\cos B}{-2(15)(9)} &= \frac{17^2 - 15^2 - 9^2}{-2(15)(9)} \\
\cos B &= \frac{17^2 - 15^2 - 9^2}{-2(15)(9)} \\
B &= \cos^{-1}\left(\frac{17^2 - 15^2 - 9^2}{-2(15)(9)}\right) \\
B &= 86.39^\circ
\end{aligned}$$

$$\begin{aligned}
15^2 &= 17^2 + 9^2 - 2(17)(9)\cos A \\
15^2 - 17^2 - 9^2 &= 17^2 + 9^2 - 2(17)(9)\cos A - 17^2 - 9^2 \\
15^2 - 17^2 - 9^2 &= -2(17)(9)\cos A \\
\frac{-2(17)(9)\cos A}{-2(17)(9)} &= \frac{15^2 - 17^2 - 9^2}{-2(17)(9)} \\
\cos A &= \frac{15^2 - 17^2 - 9^2}{-2(17)(9)} \\
A &= \cos^{-1}\left(\frac{15^2 - 17^2 - 9^2}{-2(17)(9)}\right) \\
A &= 61.72^\circ
\end{aligned}$$

$$C = 180^\circ - 61.72^\circ - 86.39^\circ = 31.89^\circ$$

3. Calculate the areas of the given triangles.

(a) Having two sides of length 11in. and 20in. with the included angle between them being 43°.

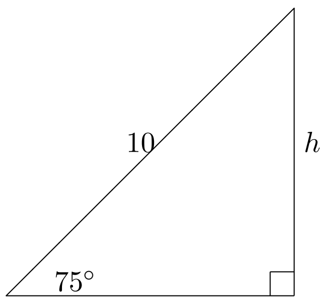
$$\text{Area} = \frac{1}{2}(11)(20)\sin 43^\circ = 75\text{in}^2$$

(b) Having sides of length 5cm, 7cm, and 9cm.

$$s = \frac{1}{2}(5 + 7 + 9) = 10.5$$

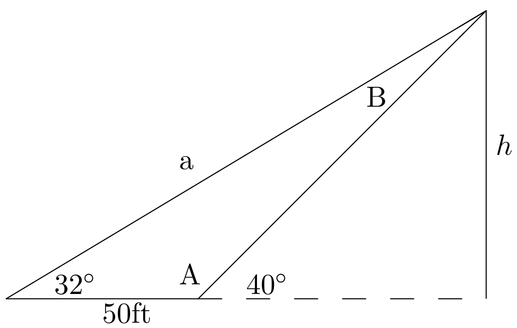
$$\text{Area} = \sqrt{10.5(10.5 - 5)(10.5 - 7)(10.5 - 9)} = 17.41\text{cm}^2$$

4. A ten foot ladder is placed against the side of a building at a 75° angle with the ground. How far up the side of the building will the top of the ladder be?



$$\begin{aligned} \sin 75^\circ &= \frac{h}{10} \\ h &= 10 \sin 75^\circ \\ h &= 9.66\text{ft} \end{aligned}$$

5. Joe measures the angle of elevation to the top of his apartment building from a point on the street below at 40°. He then paces of 50 feet in a direct line away from the building and makes a second measurement, which he finds to be 32°. Find the height of the building.



$$A = 180^\circ - 40^\circ = 140^\circ$$
$$B = 180^\circ - 140^\circ - 32^\circ = 8^\circ$$

We use this information and the law of sines to find side a.

$$\begin{aligned} \frac{\sin 140^\circ}{a} &= \frac{\sin 8^\circ}{50} \\ 50 \sin 140^\circ &= a \sin 8^\circ \\ a &= 50 \frac{\sin 140^\circ}{\sin 8^\circ} \\ a &= 230.93 \end{aligned}$$

Now we use right triangle trigonometry to find h.

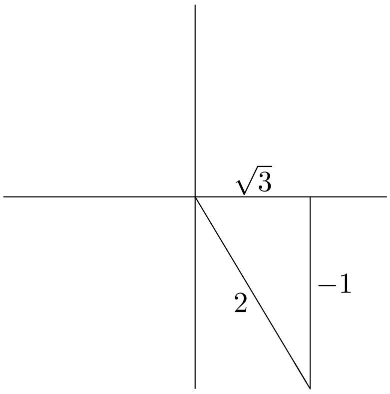
$$\sin 32^\circ = \frac{h}{230.93}$$

$$\begin{aligned} h &= 230.93 \sin 32° \\ h &= 122.37\text{ft} \end{aligned}$$

6. Find exact values for the following.

   (a) $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

   $\sin^{-1} x$ is an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Since $-\frac{1}{2}$ is negative, we can narrow this down to $-\frac{\pi}{2}$ to 0. We have an angle in quadrant IV whose sine is $-\frac{1}{2}$.

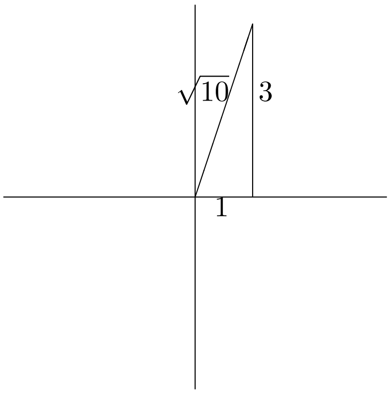


   The third side can be found using the Pythagorean theorem, but you should recognize this as the familiar 30°-60°-90° triangle. Then the cosine of this angle is

$$\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$$

   (b) $\sin\left(\tan^{-1}(3)\right)$

   $\tan^{-1} x$ is an angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Since 3 is positive, we can narrow this down to 0 to $\frac{\pi}{2}$. We have an angle in quadrant I whose tangent is $3 = \frac{3}{1}$.



   The third side can be found using the Pythagorean theorem.

$$\begin{aligned} c^2 &= 3^2 + 1^2 \\ c^2 &= 10 \\ c &= \sqrt{10} \end{aligned}$$

Then

$$\sin\left(\tan^{-1}(3)\right) = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

7. Use your calculator to evaluate $\sec^{-1}(3)$. Express your answer in degrees.

$$\begin{aligned}
\sec^{-1} 3 &= \theta \\
\sec\theta &= 3 \\
\frac{1}{\cos\theta} &= 3 \\
1 &= 3\cos\theta \\
\cos\theta &= \frac{1}{3} \\
\theta &= \cos^{-1}\left(\frac{1}{3}\right) \\
\theta &= 70.53^\circ
\end{aligned}$$